Indian Statistical Institute

MSQE | Paper II (PEB)

Past Year Papers | 2006 – 2026

prepared by

Econschool

amit@econschool.in

Updated: June 3, 2026

Contents

ISI PEB 2006

Question 1

(a) There are two sectors producing the same commodity. Labour is perfectly mobile between these two sectors. Labour market is competitive and the representative firm in each of the two sectors maximizes profit. If there are 100 units of labour and the production function for sector i is: $F(L_i) = 15\sqrt{L_i}, i = 1, 2$, find the allocation of labour between the two sectors.

(b) Suppose that prices of all variable factors and output double. What will be its effect on the short-run equilibrium output of a competitive firm? Examine whether the short-run profit of the firm will double.

(c) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750 . Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250 . Fill in, with adequate explanation, the following chart:

Year	GDP =	Consumption +	Investment +	Export –	Import
1	________	________	________	________	________
2	________	________	________	________	________

Question 2

A price-taking farmer produces a crop with labour L as the only input. His production function is: $F(L) = 10\sqrt{L} - 2L$. He has 4 units of labour in his family and he cannot hire labour from the wage labour market. He does not face any cost of employing family labour.

(a) Find out his equilibrium level of output.

(b) Suppose that the government imposes an income tax at the rate of 10 per cent. How does this affect his equilibrium output?

(c) Suppose an alternative production technology given by: $F(L) = 11\sqrt{L} - L - 15$ is available. Will the farmer adopt this alternative technology? Briefly justify your answer.

Question 3

Suppose a monopolist faces two types of consumers. In type I there is only one person whose demand for the product is given by : $Q_I = 100 - P$, where P represents price of the good. In type II there are n persons, each of whom has a demand for one unit of the good and each of them wants to pay a maximum of Rs. 5 for one unit. Monopolist cannot price discriminate between the two types. Assume that the cost of production for the good is zero. Does the equilibrium price depend on n ? Give reasons for your answer.

Question 4

The utility function of a consumer is: $U(x, y) = xy$. Suppose income of the consumer

(M) is 100 and the initial prices are $P_x = 5, P_y = 10$. Now suppose that P_x goes up to $10, P_y$ and M remaining unchanged. Assuming Slutsky compensation scheme, estimate price effect, income effect and substitution effect.

Question 5

Consider an $IS - LM$ model for a closed economy. Private consumption depends on disposable income. Income taxes (T) are lump-sum. Both private investment and speculative demand for money vary inversely with interest rate (r). However, transaction demand for money depends not on income (y) but on disposable income (y_d). Argue how the equilibrium values of private investment, private saving, government saving, disposable income and income will change, if the government raises T.

Question 6

An individual enjoys bus ride. However, buses emit smoke which he dislikes. The individual's utility function is: $U = U(x, s)$, where x is the distance (in km) traveled by bus and s is the amount of smoke consumed from bus travel.

(a) What could be the plausible alternative shapes of indifference curve between x and s?

(b) Suppose, smoke consumed from bus travel is proportional to the distance traveled: $s = \alpha x$(α is a positive parameter). Suppose further that the bus fare per km is p and that the individual has money income M to spend on bus travel. Show the budget set of the consumer in an (s, x) diagram.

(c) What can you say about an optimal choice of the individual? Will he necessarily exhaust his entire income on bus travel?

Question 7

(a) Suppose the labour supply (l) of a household is governed by maximization of its utility $(u) : u = c^{2/3}h^{1/3}$, where c is the household's consumption and h is leisure enjoyed by the household (with $h + l = 24$). Real wage rate (w) is given and the household consumes the entire labour income (wl). What is the household's labour supply? Does it depend on w?

(b) Consider now a typical Keynesian (closed) economy producing a single good and having a single household. There are two types of final expenditure - viz., investment autonomously given at 36 units and household consumption (c) equalling the household's labour income (wl). It is given that $w = 4$. Firms produce aggregate output (y) according to the production function: $y = 24\sqrt{l}$. Find the equilibrium level of output and employment. Is there any involuntary unemployment? If so, how much?

Question 8

Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta} - 1}{1-\theta}, \quad 0 < \theta < 1, \rho > 0$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

(a) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

(b) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

(c) Find an expression for s as a function of w and r.

(d) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

(e) Give an intuitive explanation of your finding in (d)

Question 9

A consumer consumes only two commodities x_1 and x_2. Suppose that her utility function is given by $U(x_1, x_2) = \min(2x_1, x_2)$.

(a) Draw a representative indifference curve of the consumer.

(b) Suppose the prices of the commodities are Rs.5 and Rs.10 respectively while the consumer's income is Rs. 100. What commodity bundle will the consumer purchase?

(c) Suppose the price of commodity 1 now increases to Rs. 8 . Decompose the change in the amount of commodity 1 purchased into income and substitution effects.

Question 10

A price taking firm makes machine tools Y using labour and capital according to the production function $Y = K^{0.25}L^{0.25}$. Labour can be hired at the beginning of every week while capital can be hired only at the beginning of every month. Let one month be considered as long run period and one week as short run period. Further assume that one month equals four weeks. The wage rate per week and the rental rate of capital per month are both 10 .

(a) Given the above information, find the short run and the long run cost functions of the firm.

(b) At the beginning of the month of January, the firm is making long run decisions given that the price of machine tools is 400. What is the long run profit maximizing number of machine tools? How many units of labour and capital should the firm hire at the beginning of January?

Question 11

Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

ISI PEB 2007

Question 1

(a) There is a cake of size 1 to be divided between two persons, 1 and 2 . Person 1 is going to cut the cake into two pieces, but person 2 will select one of the two pieces for himself first. The remaining piece will go to 1st person. What is the optimal cutting decision for player 1? Justify your answer.

(b) Kamal has been given a free ticket to attend a classical music concert. If Kamal had to pay for the ticket, he would have paid up to Rs. 300/− to attend the concert. On the same evening, Kamal's alternative entertainment option is a film music and dance event for which tickets are priced at Rs. 200/− each. Suppose also that Kamal is willing to pay up to Rs. X to attend the film music and dance event. What does Kamal do, i.e., does he attend the classical music concert, or does he attend the film music and dance show, or does he do neither? Justify your answer.

Question 2

Suppose market demand is described by the equation P = 300 − Q and competitive conditions prevail. The short-run supply curve is P = −180 + 5Q. Find the initial short-run equilibrium price and quantity. Let the long-run supply curve be P = 60 + 2Q. Verify whether the market is also in the long-run equilibrium at the initial short-run equilibrium that you have worked out. Now suppose that the market demand at every price is doubled. What is the new market demand curve? What happens to the equilibrium in the very short-run? What is the new short-run equilibrium? What is the new long-run equilibrium? If a price ceiling is imposed at the old equilibrium, estimate the perceived shortage. Show all your results in a diagram.

Question 3

(a) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750 . Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250 . Fill in, with adequate explanation, the following chart:

Year	GDP =	Consumption +	Investment +	Export −	Import
1	______	______	______	______	______
2	______	______	______	______	______

(b) Consider an IS-LM model for a closed economy with government where investment (I) is a function of rate of interest (r) only. An increase in government expenditure is found to crowd out 50 units of private investment. The government wants to prevent this by a minimum change in the supply of real money balance. It is given that $\frac{dI}{dr} = -50$, slope of the LM curve, $\frac{dr}{dy}(LM) = \frac{1}{250}$, slope of the IS curve, $\frac{dr}{dy}(IS) = -\frac{1}{125}$, and all relations are linear. Compute the change in y from the initial to the final equilibrium when all adjustments have been made.

Question 4

(a) Consider a consumer with income W who consumes three goods, which we denote as $i = 1, 2, 3$. Let the amount of good i that the consumer consumes be x_i and the price of good i be p_i. Suppose that the consumer's preference is described by the utility function $U(x_1, x_2, x_3) = x_1\sqrt{x_2 x_3}$.

i. Set up the utility maximization problem and write down the Lagrangian.

ii. Write down the first order necessary conditions for an interior maximum and then obtain the Marshallian (or uncompensated) demand functions.

(b) The production function, $Y = F(K, L)$, satisfies the following properties:

- CRS,
- symmetric in terms of inputs and
- $F(1, 1) = 1$.

The price of each input is Rs. $2/-$ per unit and the price of the product is Rs. 3/- per unit. Without using calculus find the firm's optimal level of production.

Question 5

(a) A monopolist has contracted to sell as much of his output as he likes to the government at Rs.100/- per unit. His sale to the government is positive. He also sells to private buyers at Rs150/$-$ per unit. What is the price elasticity of demand for the monopolist's products in the private market?

(b) Mrs. Pathak is very particular about her consumption of tea. She always takes 50 grams of sugar with 20 grams of ground tea. She has allocated Rs 55 for her spending on tea and sugar per month. (Assume that she doesn't offer tea to her guests or anybody else and she doesn't consume sugar for any other purpose). Sugar and tea are sold at 2 paisa per 10 grams and 50 paisa per 10 grams respectively. Determine how much of tea and sugar she demands per month.

(c) Consider the IS-LM model with government expenditure and taxation. A change in the income tax rate changes the equilibrium from $(y = 3000, r = 4\%)$ to $(y = 3500, r = 6\%)$, where y, r denote income and rate of interest, respectively. It is given that a unit increase in y increases demand for real money balance by 0.25 of a unit. Compute the change in real money demand that results from a 1% increase in the rate of interest. (Assume that all relationships are linear.)

Question 6

(a) An economy produces two goods, corn and machine, using for their production only labor and some of the goods themselves. Production of one unit of corn requires 0.1 units of corn, 0.3 machines and 5 man-hours of labor. Similarly, production of one machine requires 0.4 units of corn, 0.6 machines and 20 man-hours of labor.

i. If the economy requires 48 units of corn but no machine for final consumption, how much of each of the two commodities is to be produced? How much labor will be required?

ii. If the wage rate is Rs. 2/− per man-hour, what are the prices of corn and machines, if price of each commodity is equated to its average cost of production?

(b) Consider two consumers A and B, each with income W. They spend their entire budget over the two commodities, X and Y. Compare the demand curves of the two consumers under the assumption that their utility functions are $U_A = x + y$ and $U_B = x^2 + y^2$ respectively

Question 7

Consider a Simple Keynesian Model without government for an open economy, where both consumption and import are proportional functions of income (Y). Suppose that average propensities to consume and import are 0.8 and 0.3, respectively. The investment (I) function and the level of export (X) are given by $I = 100 + 0.4Y$ and $X = 100$.

(a) Compute the aggregate demand function if the maximum possible level of imports is 450. Can there be an equilibrium for this model? Show your result graphically.

(b) How does your answer to part a change if the limit to import is raised to 615? What can you say about the stability of equilibrium if it exists?

Question 8

Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta} - 1}{1-\theta}, \quad 0 < \theta < 1, \rho > 0$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

(a) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

(b) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

(c) Find an expression for s as a function of w and r.

(d) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

(e) Give an intuitive explanation of your finding in d

Question 9

Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

ISI PEB 2008

Question 1

There are two individuals A and B and two goods X and Y. The utility functions of A and B are given by $U_A = X_A$ and $U_B = X_B^2 + Y_B^2$ respectively where X_i, Y_i are consumption levels of the two goods by individual $i, i = A, B$.

(a) Draw the indifference curves of A and B.

(b) Suppose A is endowed with 10 units of Y and B with 10 units of X. Indicate the endowment point in a box diagram.

(c) Draw the set of Pareto optimal allocation points in the box diagram.

Question 2

Suppose an economy's aggregate output (Y) is given by the following production function: $Y = UN^\alpha, (0 < \alpha < 1)$ where U, a random variable, represents supply shock. Employment of labour (N) is determined by equating its marginal product to $\frac{W}{P}$, where W is nominal wage and P is price level. Use the notations: $u = \log \alpha + \frac{1}{\alpha} \log U; p = \log P; w = \log W$ and $y = \log Y$.

(a) Obtain the aggregate supply function (y) in terms of p, w, and u.

(b) Add the following relations: Wages are indexed: $w = \theta p, (0 \leq \theta \leq 1)$ Aggregate demand: $y = m - p, (m =$ logarithm of money, a policy variable) Find the solution of y in terms of m and u.

(c) Does monetary policy affect output

   i. if indexation is partial $(0 < \theta < 1)$,

   ii. indexation is full $(\theta = 1)$?

(d) Does the real shock affect output more when indexation is higher? Explain.

Question 3

Two firms 1 and 2 sell a single, homogeneous, infinitely divisible good in a market. Firm 1 has 40 units to sell and firm 2 has 80 units to sell. Neither firm can produce any more units. There is a demand curve: $p = a - q$, where q is the total amount placed by the firms in the market. So if q_i is the amount placed by firm $i, q = q_1 + q_2$ and p is the price that emerges. a is positive and a measure of market size. It is known that a is either 100 or 200 . The value of a is observed by both firms. After they observe the value of a, each firm decides whether or not to destroy a part of its output. This decision is made simultaneously and independently by the firms. Each firm faces a constant per unit cost of destruction equal to 10 . Whatever number of units is left over after destruction is sold by the firm in the market. Show that a firm's choice about the amount it wishes to destroy is independent of the amount chosen by the other firm. Show also that the amount destroyed by firm 2 is always positive, while firm 1 destroys a part of its output if and only if $a = 100$.[1]

Question 4

[1] The wording of this problem is not correct.

(a) Two commodities, X and Y, are produced with identical technology and are sold in competitive markets. One unit of labour can produce one unit of each of the two commodities. Labour is the only factor of production; and labour is perfectly mobile between the two sectors. The representative consumer has the utility function: $U = \sqrt{XY}$; and his income is Rs. 100/-. If 10 units of labour are available, find out the equilibrium wage in the competitive labour market.

(b) Consider an economy producing a single good by a production function

$$Y = \min\{K, L\}$$

where Y is the output of the final good. K and L are input use of capital and labour respectively. Suppose this economy is endowed with 100 units of capital and labour supply L_s is given by the function $L_s = 50w$ where w is the wage rate. Assuming that all markets are competitive find the equilibrium wage and rental rate.

Question 5

The following symbols are used: Y = output, N = employment, W = nominal wage, P = price level, P^c = expected price level. The Lucas supply function is usually written as:

$$\log Y = \log Y^* + \lambda \left(\log P - \log P^e\right)$$

where Y^* is the natural level of output. Consider an economy in which labour supply depends positively on the expected real wage:

$$\frac{W}{P^e} = N^\sigma, (\sigma > 0) \quad \text{(labour supply)}$$

Firms demand labour up to the point where its marginal product equals the given (actual) real wage $\left(\frac{W}{P}\right)$ and firm's production function is: $Y = N^\alpha, (0 < \alpha < 1)$

(a) Find the labour demand function.

(b) Equate labour demand with labour supply to eliminate W. You will get an expression involving P, P^e and N. Derive the Lucas supply function in the form given above and find the expressions for λ and Y^*.

(c) How is this type of model referred to in the literature? Explain

Question 6

Consider an IS - LM model given by the following equations

$$\begin{aligned}
C &= 200 + .5Y_D \\
I &= 150 - 1000r \\
T &= 200 \\
G &= 250 \\
\left(\tfrac{M}{P}\right)^d &= 2Y - 4000i \\
\left(\tfrac{M}{P}\right)^s &= 1600 \\
i &= r - \Pi^e
\end{aligned}$$

where C is consumption, Y_D is disposable income, I is investment, r is real rate of interest, i is nominal rate of interest, T is tax, G is government expenditure, $\left(\frac{M}{P}\right)^d$ and $\left(\frac{M}{P}\right)^s$ are real money demand and real money supply respectively and and Π^e is the expected rate of inflation. The current price level P remains always rigid.

(a) Assuming that $\Pi^e = 0$, i.e., the price level is expected to remain unchanged in future, determine the equilibrium levels of income and the rates of interest.

(b) Suppose there is a temporary increase in nominal money supply by 2%. Find the new equilibrium income and the rates of interest.

(c) Now assume that the 2% increase in nominal money supply is permanent leading to a 2% increase in the expected future price level. Work out the new equilibrium income and the rates of interest.

Question 7

A firm is contemplating to hire a salesman who would be entrusted with the task of selling a washing machine. The hired salesman is efficient with probability 0.25 and inefficient with probability 0.75 and there is no way to tell, by looking at the salesman, if he is efficient or not. An efficient salesman can sell the washing machine with probability 0.8 and an inefficient salesman can sell the machine with probability 0.4 . The firm makes a profit of Rs. 1000 if the machine is sold and gets nothing if it is not sold. In either case, however, the salesman has to be paid a wage of Rs. 100 .

(a) Calculate the expected profit of the firm.

(b) Suppose instead of a fixed payment, the firm pays a commission of $t\%$ on its profit to the salesman (i.e., if the good is sold the salesman gets Rs. $1000 \times \frac{t}{100}$ and nothing if the good remains unsold). A salesman, irrespective of whether he is efficient or inefficient, has an alternative option of working for Rs.80. A salesman knows whether he is efficient or not and cares only about the expected value of his income: find the value of t that will maximize the expected profit of the firm.

Question 8

(a) On a tropical island there are 100 boat builders, numbered 1 through 100 . Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i, from 1 to 100 , boat builder has a cost function $C_i(y) = 11 + iy$. Assume that in the cost function the fixed cost, 11, is a quasi-fixed cost, that is, it is only paid if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

(b) Consider the market for a particular good. There are two types of customers: those of type 1 are the low demand customers, each with a demand function of the form $p = 10 - q_1$, and those of type 2, who are the high demand customers, each with a demand function of the form $p = 2\left(10 - q_2\right)$. The firm producing the product is monopolist in this market and has a cost function $C(q) = 4q^2$ where $q = q_1 + q_2$

i. Suppose the firm is unable to prevent the customers from selling the good to one another, so that the monopolist cannot charge different customers different prices. What prices per unit will the monopolist charge to maximize its total profit and what will be the equilibrium quantities to be supplied to the two groups in equilibrium?

ii. Suppose the firm realizes that by asking for IDs it can identify the types of the customers (for instance, type 1 's are students who can be identified using their student IDs). It can thus charge different per unit prices to the two groups, if it is optimal to do so. Find the profit maximizing prices to be charged to the two groups.

Question 9

Consider the following box with 16 squares:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

There are two players 1 and 2, and the game begins with player 1 selecting one of the boxes marked 1 to 16. Following such a selection, the selected box, as well as all boxes in the square of which the selected box constitutes the leftmost and lowest corner, will be deleted. For example, if he selects box 7, then all the boxes, 3,4,7 and 8 are deleted. Similarly, if he selects box 9, then all boxes 1 to 12 are deleted. Next it is player 2 's turn to select a box from the remaining boxes. The same deletion rule applies in this case. It is then player 1's turn again, and so on. Whoever deletes the last box loses the game? What is a winning strategy for player 1 in this game?

Question 10

(a) Mr. A's yearly budget for his car is Rs. 100,000 , which he spends completely on petrol (P) and on all other expenses for his car (M). All other expenses for car (M) is measured in Rupees, so you can consider that price of M is Re. 1. When price of petrol is Rs. 40 per liter, Mr. A buys 1,000 liters per year. The petrol price rises to Rs. 50 per liter, and to offset the harm to Mr. A, the government gives him a cash transfer of Rs. 10,000 per year.

i. Write down Mr. A's yearly budget equation under the 'price rise plus transfer' situation.

ii. What will happen to his petrol consumption - increase, decrease, or remain the same?

iii. Will he be better or worse off after the price rise plus transfer than he was before? [Do not refer to any utility function or indifference curves to answer]

(b) Mr. B earns Rs. 500 today and Rs. 500 tomorrow. He can save for future by investing today in bonds that return tomorrow the principal plus the interest. He can also borrow from his bank paying an interest. When the interest rates on both bank loans and bonds are 15%Mr. B chooses neither to save nor to borrow.

i. Suppose the interest rate on bank loans goes up to 30% and the interest rate on bonds fall to 5%. Write down the equation of the new budget constraint and draw his budget line.

ii. Will he lend or borrow? By how much?

ISI PEB 2009

Question 1

Consider the following model of the economy:

$$C = c_0 + c_1 Y_D$$
$$T = t_0 + t_1 Y$$
$$Y_D = Y - T$$

C denotes consumption, $c_0 > 0$ denotes autonomous consumption, $0 < c_1 < 1$ is the marginal propensity to consume, Y, denotes income, T denotes taxes, Y_D denotes disposable income and $t_0 > 0, t_1 > 0$. Assume a closed economy where government spending G, and investment I, are exogenously given by $\bar{G}$ and $\bar{I}$ respectively.

(a) Interpret t_1 in words. Is it greater or less than 1? Explain your answer.

(b) Solve for equilibrium output, Y^*.

(c) What is the multiplier? Does the economy respond more to changes in autonomous spending (such as changes in c_0, $\bar{G}$, and, $\bar{I}$) when t_1 is zero or when t_1 is positive ? Explain.

Question 2

Consider an agent who values consumption in periods 0 and 1 according to the utility function

$$u(c_0, c_1) = \log c_0 + \delta \log c_1$$

where $0 < \delta < 1$. Suppose that the agent has wealth ω in period 0 of which she can save any portion in order to consume in period 1. If she saves Re. 1, she is paid interest r so that her budget constraint is

$$c_0 + \frac{c_1}{1+r} = \omega$$

(a) Derive the agent's demand for c_0 and c_1 as a function of r and ω.

(b) What happens to c_0 and c_1 as r increases? Interpret.

(c) For what relationship between ω and r will she consume the same amount in both periods?

Question 3

Consider a firm with production function $F(x_1, x_2) = \min(2x_1, x_1 + x_2)$ where x_1 and x_2 are amounts of factors 1 and 2 .

(a) Draw an isoquant for output level 10 .

(b) Show that the production function exhibits constant returns to scale.

(c) Suppose that the firm faces input prices $w_1 = w_2 = 1$. What is the firms' cost function?

Question 4

Consider an exchange economy consisting of two individuals 1 and 2, and two goods X and Y. The utility function of individual i, $U_i = X_i + Y_i$. Individual 1 has 3 units of X and 7 units of Y to begin with. Similarly, individual 2 has 7 units of X and 3 units of Y to begin with.

(a) What is the set of Pareto optimal outcomes in this economy? Justify your answer

(b) What is the set of perfectly competitive (Walrasian) outcomes? You may use diagrams for parts (i) and (ii).

(c) Are the perfectly competitive outcomes Pareto optimal? Does this result hold generally in all exchange economies?

Question 5

A monopoly sells its product in two separate markets. The inverse demand function in market 1 is given by $q_1 = 10 - p_1$, and the inverse demand function in market 2 is given by $q_2 = a - p_2$, where $10 < a \leq 20$. The monopolist's cost function is $C(q) = 5q$, where q is aggregate output.

(a) Suppose the monopolist must set the same price in both markets. What is its optimal price? What is the reason behind the restriction that $a \leq 20$?

(b) Suppose the monopolist can charge different prices in the two markets. Compute the prices it will set in the two markets.

(c) Under what conditions does the monopolist benefit from the ability to charge different prices?

(d) Compute consumers' surplus in cases a and b. Who benefits from differential pricing and who does not relative to the case where the same price is charged in both markets?

Question 6

Consider an industry with 3 firms, each having marginal cost equal to 0. The inverse demand curve facing this industry is $p = 120 - q$, where q is aggregate output.

(a) If each firm behaves as in the Cournot model, what is firm 1 's optimal output choice as a function of its beliefs about other firms' output choices?

(b) What output do the firms produce in equilibrium?

(c) Firms 2 and 3 decide to merge and form a single firm with marginal cost still equal to 0. What output do the two firms produce in equilibrium? Is firm 1 better off as a result? Are firms 2 and 3 better off post-merger? Would it be better for all the firms to form a cartel instead? Explain in each case.

Question 7

Suppose the economy's production function is given by

$$Y_t = 0.5\sqrt{K_t}\sqrt{N_t} \tag{1}$$

Y_t denotes output, K_t denotes the aggregate capital stock in the economy, and N denotes the number of workers (which is fixed). The evolution of the capital stock is given by;

$$K_{t+1} = sY_t + (1 - \delta)K_t \tag{2}$$

where the savings rate of the economy is denoted by, s, and the depreciation rate is given by, δ.

(a) Using equation (2), show that the change in the capital stock per worker, $\frac{K_{t+1}-K_t}{N}$, is equal to savings per worker minus depreciation per worker.

(b) Derive the economy's steady state levels of $\frac{K}{N}$ and $\frac{Y}{N}$ in terms of the savings rate and the depreciation rate.

(c) Derive the equation for the steady state level of consumption per worker in terms of the savings rate and the depreciation rate.

(d) Is there a savings rate that is optimal, i.e., maximizes steady state consumption per worker ? If so, derive an expression for the optimal savings rate. Using words and graphs, discuss your answer.

Question 8

Suppose there are 10 individuals in a society, 5 of whom are of high ability, and 5 of low ability. Individuals know their own abilities. Suppose that each individual lives for two periods and is deciding whether or not to go to college in period 1. When individuals make decisions in period 1, they choose that option which gives the highest lifetime payoff, i.e., the sum of earnings and expenses in both periods. Education can only be acquired in period 1. In the absence of schooling, high and low ability individuals can earn y_H and y_L respectively in each period. With education, period 2 earning increases to $(1 + a)y_H$ for high ability types and $(1 + a)y_L$ for low ability types. Earnings would equal 0 in period 1 if an individual decided to go to college in that period. Tuition fee for any individual is equal to T. Assume y_H and y_L are both positive, as is T.

(a) Find the condition that determines whether each type of person will go to college in period 1. What is the minimum that a can be if it is to be feasible for any type of individual to acquire education?

(b) Suppose $y_H = 50, y_L = 40, a = 3$. For what values of T will a high ability person go to college? And a low ability person? Which type is more likely to acquire education?

(c) Now assume the government chooses to subsidise education by setting tuition equal to 60. What happens to educational attainment?

(d) Suppose now to pay for the education subsidy, the government decides to impose a x% tax on earnings in any period greater than 50. So if an individual earns 80 in a period, he would pay a tax in that period equal to x% of 30 . The government wants all individuals to acquire education, and also wants to cover the cost of the education subsidy in period 1 through tax revenues collected in both periods. What value of x should the government set?

Question 9

Consider the goods market with exogenous (constant) investment $\bar{I}$, exogenous government spending, $\bar{G}$ and constant taxes, T. The consumption equation is given by,

$$C = c_0 + c_1(Y - T)$$

where C denotes consumption, c_0 denotes autonomous consumption, and c_1 the marginal propensity to consume.

(a) Solve for equilibrium output. What is the value of the multiplier?

(b) Now let investment depend on Y and the interest rate, i

$$I = b_0 + b_1 Y - b_2 i$$

where b_0 and b_1 are parameters. Solve for equilibrium output. At a given interest rate, is the effect of an increase in autonomous spending bigger than it was in part a? In answering this, assume that $c_1 + b_1 < 1$.

(c) Now, introduce the financial market equilibrium condition

$$\frac{M}{P} = d_1 Y - d_2 i$$

where $\frac{M}{P}$ denotes the real money supply. Derive the multiplier. Assume that investment is given by the equation in part b.

(d) Is the multiplier you obtained in part c smaller or larger than the multiplier you obtained in part a. Explain how your answer depends on the behavioral equations for consumption, investment, and money demand.

Question 10

(a) A college is trying to fill one remaining seat in its Masters programme. It judges the merit of any applicant by giving him an entrance test. It is known that there are two interested applicants who will apply sequentially. If the college admits the first applicant, it cannot admit the second. If it rejects the first applicant, it must admit the second. It is not possible to delay a decision on the first applicant till the second applicant is tested. At the time of admitting or rejecting the first applicant, the college thinks the second applicant's mark will be a continuous random variable drawn from the uniform distribution between 0 and 100. (Recall that a random variable x is uniformly distibuted on $[a, b]$ if the density function of x is given by $f(x) = \frac{1}{b-a}$ for $x \in [a, b]$). If the college wants to maximize the expected mark of its admitted student, what is the lowest mark for which it should admit the first applicant?

(b) Now suppose there are three applicants who apply sequentially. Before an applicant is tested, it is known that his likely mark is an independent continuous random variable drawn from the uniform distribution between 0 and 100. What is the lowest mark for which the college should admit the first student? What is the lowest mark for which the college should admit the second student in case the first is rejected?

ISI PEB 2010

Question 1

(a) Imagine a closed economy in which tax is imposed only on income. The government spending (G) is required (by a balanced budget amendment to the relevant law) to be equal to the tax revenue; thus $G = tY$, where t is the tax rate and Y is income. Consumption expenditure (C) is proportional to disposable income and investment (I) is exogenously given.

i. Explain why government spending is endogenous in this model.

ii. Is the multiplier in this model larger or smaller than in the case in which government spending is exogenous?

iii. When t increases, does Y decrease, increase or stay the same? Give an answer with intuitive explanation.

(b) Consider the following macroeconomic model with notation having usual meanings: $C = 100 + 1.3Y$ (Consumption function), $I = \frac{500}{r}$ (Investment function), $M^D = 150Y + 100 - 1500r$ (Demand for money function) and $M^S = 2100$ (Supply of money). Do you think that there exists an equilibrium? Justify your answer using the IS-LM model.

Question 2

Consider a market with two firms. Let the cost function of each firm be $C(q) = mq$ where $q \geq 0$. Let the inverse demand functions of firms 1 and 2 be $P_1(q_1, q_2) = a - q_1 - sq_2$ and $P_2(q_1, q_2) = a - q_2 - sq_1$, respectively. Assume that $0 < s < 1$ and $a > m > 0$

(a) Find the Cournot equilibrium quantities of the two firms.

(b) Using the inverse demand functions $P_1(q_1, q_2)$ and $P_2(q_1, q_2)$, derive direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ of firms 1 and 2 .

(c) Using the direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$, find the Bertrand equilibrium prices.

Question 3

(a) A monopolist can sell his output in two geographically separated markets A and B. The total cost function is $TC = 5 + 3(Q_A + Q_B)$ where Q_A and Q_B are quantities sold in markets A and B respectively. The demand functions for the two markets are, respectively, $P_A = 15 - Q_A$ and $P_B = 25 - 2Q_B$. Calculate the firm's price, output, profit and the deadweight loss to the society if it can get involved in price discrimination.

(b) Suppose that you have the following information. Each month an airline sells 1500 business-class tickets at Rs. 200 per ticket and 6000 economy class tickets at Rs. 80 per ticket. The airline treats business class and economy class as two separate markets. The airline knows the demand curves for the two markets and maximizes profit. It is also known that the demand curve of each of the two markets is linear and marginal cost associated with each ticket is Rs. 50 .

i. Use the above information to construct the demand curves for economy class and business class tickets.

ii. What would be the equilibrium quantities and prices if the airline could not get involved in price discrimination?

Question 4

Consider an economy producing two goods 1 and 2 using the following production functions: $X_1 = L_1^{\frac{1}{2}} K^{\frac{1}{2}}$ and $X_2 = L_2^{\frac{1}{2}} T^{\frac{1}{2}}$, where X_1 and X_2 are the outputs of good 1 and 2, respectively, K is capital used in production of good 1, T is land used in production of good 2 and L_1 and L_2 are amounts of labour used in production of good 1 and 2 , respectively. Full employment of all factors is assumed implying the following: $K = \bar{K}, T = \bar{T}, L_1 + L_2 = \bar{L}$ where $\bar{K}, \bar{T}$ and $\bar{L}$ are total amounts of capital, land and labour available to the economy. Labour is assumed to be perfectly mobile between sectors 1 and 2 . The underlying preference pattern of the economy generates the relative demand function, $\frac{D_1}{D_2} = \gamma \left(\frac{p_1}{p_2}\right)^{-2}$, where D_1 and D_2 are the demands and p_1 and p_2 prices of good 1 and 2 respectively. All markets (both commodities and factors) are competitive.

(a) Derive the relationship between $\frac{X_1}{X_2}$ and $\frac{p_1}{p_2}$.

(b) Suppose that γ goes up. What can you say about the new equilibrium relative price?

Question 5

Consider the IS-LM representation of an economy with the following features:

- The economy is engaged in export and import of goods and services, but not in capital transactions with foreign countries.
- Nominal exchange rate, that is, domestic currency per unit of foreign currency, e, is flexible.
- Foreign price level (P^*) and domestic price level (P) are given exogenously.
- There is no capital mobility and e has to be adjusted to balance trade in equilibrium. The trade balance (TB) equation (with an autonomous part $T > 0$) is given by $TB = \bar{T} + \frac{\beta P^*}{P} - mY$, where Y is GDP and β and m are positive parameters, m being the marginal propensity to import.

(a) Taking into account trade balance equilibrium and commodity market equilibrium, derive the relationship between Y and the interest rate (r). Is it the same as in the IS curve for the closed economy? Explain. Draw also the LM curve on the (Y, r) plane.

(b) Suppose that the government spending is increased. Determine graphically the new equilibrium value of Y. How does the equilibrium value of e change?

(c) Suppose that P^* is increased. How does it affect the equilibrium values of Y and e?

Question 6

(a) A firm can produce its product with two alternative technologies given by $Y = \min\left\{\frac{K}{3}, \frac{L}{2}\right\}$ and $Y = \min\left\{\frac{K}{2}, \frac{L}{3}\right\}$. The factor markets are competitive and the marginal cost of production is $Rs.20$ with each of these two technologies. Find the equation of the expansion path of the firm if it uses a third production technology given by $Y = K^{\frac{2}{3}}L^{\frac{1}{3}}$.

(b) A utility maximizing consumer with a given money income consumes two commodities X and Y. He is a price taker in the market for X. For Y there are two alternatives:

- He purchases Y from the market being a price taker
- The government supplies a fixed quantity of it through ration shops free of cost.

Is the consumer necessarily better off in case (A) or (B)? Explain your answer with respect to the following cases:

i. Indifference curves are strictly convex to the origin.

ii. X and Y are perfect substitutes.

iii. X and Y are perfect complements.

Question 7

Indicate, with adequate explanations, whether each of the following statements is TRUE or FALSE.

(a) If an increase in the price of a good leads a consumer to buy more of it, then an increase in his income will lead him to buy less of the good. By the same argument, if an increase in the price of the good leads him to buy less of it, then an increase in his income will lead him to buy more of the good.

(b) Suppose that a farmer, who receives all his income from the sale of his crop at a price beyond his control, consumes more of the crop as a result of the price increase. Then the crop is a normal good.

(c) If the non-wage income of a person increases then he chooses to work less at a given wage rate. Then he will choose to work more as his wage rate increases.

(d) The amount of stipends which Indian Statistical Institute pays to its students is a part of GDP.

Question 8

Consider a Solow model with the production function $Y = K^{\frac{1}{2}}L^{\frac{1}{2}}$, where Y, K and L are levels of output, capital and labour, respectively. Suppose, 20% of income is saved and invested. Assume that the rate of growth of labour force, that is, $\left(\frac{\frac{dL}{dt}}{L}\right) = 0.05$.

(a) Find the capital-labour ratio, rate of growth of output, rate of growth of savings and the wage rate, in the steady state growth equilibrium.

(b) Suppose that the proportion of income saved goes up from 20% to 40%. What will be the new steady state growth rate of output?

(c) Is the rate of growth of output in the new steady state equilibrium different from that obtained just before attaining the new steady state (after deviating from the old steady state)? Explain.

Question 9

(a) Consider the utility function $U(x_1, x_2) = (x_1 - s_1)^{0.5}(x_2 - s_2)^{0.5}$, where $s_1 > 0$ and $s_2 > 0$ represent subsistence consumption and $x_1 \geq s_1$ and $x_2 \geq s_2$. Using the standard budget constraint, derive the budget share functions and demand functions of the utility maximizing consumer. Are they linear in prices? Justify your answer.

(b) Suppose that a consumer maximizes $U(x_1, x_2)$ subject to the budget constraint $p(x_1 + x_2) \leq M$ where $x_1 \geq 0, x_2 \geq 0, M > 0$ and $p > 0$. Moreover, assume that the utility function is symmetric, that is $U(x_1, x_2) = U(x_2, x_1)$ for all $x_1 \geq 0$ and $x_2 \geq 0$. If the solution (x_1^*, x_2^*) to the consumer's constrained optimization problem exists and is unique, then show that $x_1^* = x_2^*$.

Question 10

(a) Consider an economy with two persons (A and B) and two goods (1 and 2). Utility functions of the two persons are given by $U_A(x_{A1}, x_{A2}) = x_{A1}^{\alpha} + x_{A2}^{\alpha}$ with $0 < \alpha < 1$; and $U_B(x_{B1}, x_{B2}) = x_{B1} + x_{B2}$. Derive the equation of the contract curve and mention its properties.

(b)
  i. A firm can produce a product at a constant average (marginal) cost of Rs. 4. The demand for the good is given by $x = 100 - 10p$. Assume that the firm owner requires a profit of Rs. 80. Determine the level of output and the price that yields maximum revenue if this profit constraint is to be fulfilled.
  ii. What will be the effects on price and output if the targeted profit is increased to Rs. 100?
  iii. Also find out the effects of the increase in marginal cost from Rs. 4 to Rs. 8 on price and output.

ISI PEB 2011

Question 1

A monopolist sells two products, X and Y. There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4

The monopolist who wants to maximize total payoffs has three alternative marketing strategies:

- sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing);
- offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and
- offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy).

However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

Question 2

Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill.

(a) Find equilibrium consumption under plan A.

(b) Find equilibrium consumption under plan B.

(c) Explain your answer if the equilibrium outcome in case b differs from that in case a.

Question 3

Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ where C_t represents consumption or use of the resource at period t and $\delta (0 < \delta < 1)$ is the discount factor

(a) Set up the utility maximization problem of the community and interpret the first order condition.

(b) Express the optimal consumption C_t for any period t in terms of the parameters δ and X.

(c) If an unanticipated discovery of an additional stock of X' occurs at the beginning of period $T(0 < T < \infty)$, what will be the new level of consumption at each period from T onwards?

Question 4

A consumer, with a given money income M, consumes n goods $x_1, x_2, \ldots, x_n$ with given prices $p_1, p_2, \ldots, p_n$

(a) Suppose his utility function is $U(x_1, x_2, \ldots, x_n) = \text{Max}(x_1, x_2, \ldots, x_n)$. Find the Marshallian demand function for good x_i and draw it in a graph.

(b) Suppose his utility function is $U(x_1, x_2, \ldots, x_n) = \text{Min}(x_1, x_2, \ldots, x_n)$. Find the income and the own price elasticities of demand for good x_i

Question 5

An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, \ldots \omega_n$ of n goods. The ith individual has a utility function $U\left(C_1^i, C_2^i, \ldots C_n^i\right) = C_1^i C_2^i \ldots C_n^i$, where C_j^i is consumption of good j of individual i.

(a) Define an allocation, a Pareto inferior allocation and a Pareto optimal allocation for this economy.

(b) Find an allocation which is Pareto inferior and an allocation which is Pareto optimal.

(c) Consider an allocation where $C_j^i = \lambda^i \omega_j \forall j, \sum_i \lambda^i = 1$. Is this allocation Pareto optimal?

Question 6

Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - \frac{3}{2} q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Now, answer the following questions.

(a) How much will the monopolist sell in the domestic market and how much will it sell in the foreign market?

(b) Suppose, the home government imposes a restriction on the amount that the monopolist can sell in the foreign market. In particular, the monopolist is not allowed to sell more than $1/6$ units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

Question 7

An economy produces two goods, food (F) and manufacturing (M). Food is produced by the production function $F = (L_F)^{\frac{1}{2}}(T)^{\frac{1}{2}}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{\frac{1}{2}}(K)^{\frac{1}{2}}$, where L_M is the labour employed, K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36, T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find out the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b) Next, we introduce a small change in the description of the economy given above. Assume, everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

(c) Suggest a measure of welfare for the economy as a whole.

(d) Using the above given data and your measure of welfare, determine whether the scenario given in problem b, where land is owned by none, better or worse for the economy as a whole, compared to the scenario given in problem a, where land is owned by the landlords?

(e) What do you think is the source of the difference in welfare levels (if any) under case a and case b.

Question 8

An economy produces a single homogeneous good in a perfectly competitive set up, using the production function $Y = AF(L, K)$, where Y is the output of the good, L and K are the amount of labour and capital respectively and A is the technological productivity parameter. Further, assume that F is homogeneous of degree one in L and K. Labour and capital in this economy remains fully employed. It has also been observed that the total wage earning of this economy is equal to the total earnings of capital in the economy at all points in time. Answer the following questions.

(a) It is observed that over a given period the labour force grew by 4%, the capital stock grew by 3%, and output grew by 9%. What then was the growth rate of the technological productivity parameter (A) over that period?

(b) Over another period the wage rate of labour in this economy exhibited a growth of 30%, rental rate of capital grew by 10% and the price of the good over the same period grew by 5%. Find out the growth rate of the technological productivity parameter (A) over this period.

(c) Over yet another period, it was observed that there was no growth in the technological productivity, and the wages grew by 30% and rental rate grew by 10%. Infer from this, the growth rate of the price of the good over the period.

Question 9

An economy produces two goods $-m$ and g. Capitalists earn a total income, R (R_m from sector m plus R_g from sector g), but consumes only good m, spending a fixed proportion c of their income on it. Workers do not save. Workers in sector m spend a fixed proportion α of their income (W) on good g and the rest on good m. [However, whatever wages are paid in sector g are spent entirely for the consumption of good g only so that we ignore wages in this sector for computing both income generation therein and the expenditure made on its output.] The categories of income and expenditure in the two sectors are shown in detail in the chart below.

Sector m		Sector g	
Income generated	Expenditure on good m	Income generated (net of wages)	Expenditure on good g (net of that by own workers)
Capitalists income (R_m), Wages (W)	Capitalists consumption ($C = c.R$), Consumption of workers of sector m ($(1-\alpha)W$), Investment I	Capitalists income (R_g)	Consumption of workers of sector m (αW)

Further, investment expenditure (I), made exclusively on m -good, is autonomous and income distribution in sector m is exogenously given: $R_m = \theta.W(\theta \text{ given })$

(a) Equating aggregate income with aggregate expenditure for the economy, show that capitalists' income (R) is determined exclusively by their own expenditure (C and I). Is there any multiplier effect of I on R ? Give arguments.

(b) Show that I (along with c, α and θ) also determines W.

Question 10

Consider two countries - a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re .1 per unit and is in equilibrium initially (i.e. in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (Y_t) , but consumption expenditure in year t (C_t) is given by: $C_t = 0.5Y_{t-1}$. The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at zero interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find out income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium in each of the following two alternative cases:

(a) The country takes the loan in year 1 only.

(b) The country takes the loan in year 1 and renews it every year.

ISI PEB 2012

Question 1

A price taking firm makes machine tools Y using labour and capital according to the following production function

$$Y = L^{0.25}K^{0.25}$$

Labour can be hired at the beginning of every week, while capital can be hired only at the beginning of every month. It is given that the wage rate = rental rate of capital = 10. Show that the short run (week) cost function is $10Y^4/K^*$ where the amount of capital is fixed at K^* and the long run (month) cost function is $20Y^2$.

Question 2

Consider the following IS-LM model

$$\begin{aligned} C &= 200 + 0.25Y_D \\ I &= 150 + 0.25Y - 1000i \\ G &= 250 \\ T &= 200 \\ (m/p)^d &= 2Y - 8000i \\ (m/p) &= 1600 \end{aligned}$$

where C = aggregate consumption, I = investment, G = government expenditures, T = taxes, $(m/p)^d$ = money demand, (m/p) = money supply, Y_D = disposable income $(Y - T)$. Solve for the equilibrium values of all variables. How is the solution altered when money supply is increased to $(m/p) = 1840$? Explain intuitively the effect of expansionary monetary policy on investment in the short run.

Question 3

Suppose that a price-taking consumer A maximizes the utility function $U(x, y) = x^\alpha + y^\alpha$ with $\alpha > 0$ subject to a budget constraint. Assume prices of both goods, x and y, are equal. Derive the demand function for both goods. What would your answer be if the price of x is twice that of the price of y?

Question 4

Assume the production function for the economy is given by

$$Y = L^{0.5}K^{0.5}$$

where Y denotes output, K denotes the capital stock and L denotes labour. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ lies between 0 and 1 and is the rate of depreciation of capital. I represents investment, given by $I_t = sY_t$, where s is the savings rate. Derive the expression of steady state consumption and find out the savings rate that maximizes steady state consumption.

Question 5

There are two goods x and y. Individual A has endowments of 25 units of good x and 15 units of good y. Individual B has endowments of 15 units of good x and 30 units of good y. The price of good y is Re. 1 , no matter whether the individual buys or sells the good. The price of good x is Re. 1 if the individual wishes to sell it. It is, however, Rs. 3 if the individual wishes to buy it. Let C_x and C_y denote the consumption of these goods. Suppose that individual B chooses to consume 20 units of good x and individual A does not buy or sell any of the goods and chooses to consume her endowment. Could A and B have the same preferences?

Question 6

A monopolist has cost function $c(y) = y$ so that its marginal cost is constant at Re. 1 per unit. It faces the following demand curve $D(p) = \begin{cases} 0, \text{ if} & p > 20 \\ \frac{100}{p}, \text{ if} & p \leq 20 \end{cases}$ Find the profit maximizing level of output if the government imposes a per unit tax of Re. 1 per unit, and also the dead-weight loss from the tax.

Question 7

A library has to be located on the interval $[0, 1]$. There are three consumers A, B and C located on the interval at locations 0.3,0.4 and 0.6, respectively. If the library is located at x, then A, B and $C's$ utilities are given by $-|x-0.3|$, $-|x-0.4|$ and $-|x-0.6|$, respectively Define a Pareto-optimal location and examine whether the locations $x = 0.1, x = 0.3$ and $x = 0.6$ are Pareto-optimal or not.

Question 8

Consider an economy where the agents live for only two periods and where there is only one good. The life-time utility of an agent is given by $U = u(c) + \beta v(d)$, where u and v are the first and second period utilities, c and d are the first and second period consumptions and β is the discount factor. β lies between 0 and 1 . Assume that both u and v are strictly increasing and concave functions. In the first period, income is w and in the second period, income is zero. The interest rate on savings carried from period 1 to period 2 is r. There is a government that taxes first period income. A proportion τ of income is taken away by the government as taxes. This is then returned in the second period to the agent as a lump sum transfer T. The government's budget is balanced i.e., $T = \tau w$. Set up the agent's optimization problem and write the first order condition assuming an interior solution. For given values of r, β, w, show that increasing T will reduce consumer utility if the interest rate is strictly positive.

Question 9

A monopolist sells two products, X and Y. There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4

The monopolist who wants to maximize total payoffs has three alternative marketing strategies:

- sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing);
- offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and
- offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy).

However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

Question 10

Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan B.

Question 11

Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum \delta^t \ln(C_t)$ where C_t represents consumption or use of the resource at period t and $\delta (0 < \delta < 1)$ is the discount factor. Express the optimal consumption C_t for any period t in terms of the parameter δ and X.

Question 12

A consumer, with a given money income M, consumes 2 goods x_1 and x_2 with given prices p_1 and p_2. Suppose that his utility function is $U(x_1, x_2) = \text{Max}(x_1, x_2)$. Find the Marshallian demand function for goods x_1, x_2 and draw it in a graph. Further, suppose that his utility function is $U(x_1, x_2) = \text{Min}(x_1, x_2)$. Find the income and the own price elasticities of demand for goods x_1 and x_2.

Question 13

An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, \ldots, \omega_n$ of n goods. The ith individual has a utility function $U\left(C_1^i, C_2^i, \ldots C_n^i\right) = C_1^i C_2^i \ldots C_n^i$, where C_j^i is consumption of good j of individual i

(a) Define an allocation,a Pareto inferior allocation and a Pareto optimal allocation for this economy.

(b) Consider an allocation where $C_j^i = \lambda^i \omega_j$ for all j, $\sum_i \lambda^i = 1$. Is this allocation Pareto optimal?

Question 14

Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - \left(\frac{3}{2}\right) q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Suppose, that the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

Question 15

An economy produces two goods, food (F) and manufacturing (M). Food is produced by the production function $F = (L_F)^{1/2} (T)^{1/2}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{1/2} (K)^{1/2}$, where L_M is the labour employed, K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36, T = 49$ and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b) Next, we introduce a small change in the description of the economy given above. Assume that everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

Question 16

Consider two countries - a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re. 1 per unit and is in equilibrium initially (i.e., in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (say , Y_t) , but consumption expenditure in year t (say, C_t) is given by: $C_t = 0.5Y_{t-1}$. The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at zero interest rate, but on conditions that it be

- used for investment only and
- repaid in full at the beginning of the next year.

The loan may be renewed every year, but on the same conditions as above. Find the income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium when the country takes the loan in year 1 only.

ISI PEB 2013

Question 1

An agent earns w units of wage while young, and earns nothing while old. The agent lives for two periods and consumes in both the periods. The utility function for the agent is given by $u = \log c_1 + \log c_2$, where c_i is the consumption in period $i = 1, 2$. The agent faces a constant rate of interest r (net interest rate) at which it can freely lend or borrow,

(a) Find out the level of saving of the agent while young.

(b) What would be the consequence of a rise in the interest rate, r on the savings of the agent?

Question 2

Consider a city that has a number of fast food stalls selling Masala Dosa (MD). All vendors have a marginal cost of Rs. 15/− per MD, and can sell at most 100MD a day.

(a) If the price of an MD is Rs. 20/−, how much does each vendor want to sell?

(b) If demand for MD be $d(p) = 4400 - 120p$, where p denotes price per MD, and each vendor sells exactly 100 units of MD, then how many vendors selling MD are there in the market?

(c) Suppose that the city authorities decide to restrict the number of vendors to 20. What would be the market price of MD in that case?

(d) If the city authorities decide to issue permits to the vendors keeping the number unchanged at 20, what is the maximum that a vendor will be willing to pay for obtaining such a permit?

Question 3

A firm is deciding whether to hire a worker for a day at a daily wage of Rs. 20/−. If hired, the worker can work for a maximum of 10 hours during the day. The worker can be used to produce two intermediate inputs, 1 and 2, which can then be combined to produce a final good. If the worker produces only 1, then he can produce 10 units of input 1 in an hour. However, if the worker produces only 2, then he can produce 20 units of input 2 in an hour. Denoting the levels of production of the amount produced of the intermediate goods by k_1 and k_2, the production function of the final good is given by $\sqrt{k_1 k_2}$. Let the final product be sold at the end of the day at a per unit price of Rs. 1/− Solve for the firms optimal hiring, production and sale decision.

Question 4

A monopolist has contracted with the government to sell as much of its output as it likes to the government at Rs. 100/ - per unit. Its sales to the government are positive, and it also sells its output to buyers at Rs. 150/ - per unit. What is the price elasticity of demand for the monopolists services in the private market?

Question 5

Consider the following production function with usual notations.

$$Y = K^{\alpha}L^{1-\alpha} - \beta K + \theta L \text{ with } 0 < \alpha < 1, \beta > 0, \theta > 0$$

Examine the validity of the following statements.

(a) Production function satisfies constant returns to scale.

(b) The demand function for labour is defined for all non-negative wage rates.

(c) The demand function for capital is undefined when price of capital service is zero.

Question 6

Suppose that due to technological progress labour requirement per unit of output is halved in a Simple Keynesian model where output is proportional to the level of employment. What happens to the equilibrium level of output and the equilibrium level of employment in this case? Consider a modified Keynesian model where consumption expenditure is proportional to labour income and wage-rate is given. Does technological progress produce a different effect on the equilibrium level of output in this case?

Question 7

A positive investment multiplier does not exist in an open economy simple Keynesian model when the entire amount of investment goods is supplied from import. Examine the validity of this statement.

Question 8

A consumer consumes two goods, x_1 and x_2, with the following utility function

$$U(x_1, x_2) = U_1(x_1) + U_2(x_2)$$

Suppose that the income elasticity is positive. It is claimed that in the above set-up all goods are normal. Prove or disprove this claim.

Question 9

A consumer derives his market demand, say x, for the product X as $x = 10 + \frac{m}{10p_x}$, where $m > 0$ is his money income and p_x is the price per unit of X. Suppose that initially he has money income $m = 120$, and the price of the product is $p_x = 3$. Further, the price of the product is now changed to $p'_x = 2$. Find the price effect. Then decompose price effect into substitution effect and income effect.

Question 10

Consider an otherwise identical Solow model of economic growth where the entire income is consumed.

(a) Analyse how wage and rental rate on capital would change over time.

(b) Can the economy attain steady state equilibrium?

ISI PEB 2014

Question 1

Consider a firm that can sell in the domestic market where it is a monopolist, and/or in the export market. The domestic demand is given by $p = 10 - q$, and export price is 5. Suppose the firm has a constant marginal cost of 4 and a capacity constraint on output of 100.

(a) Solve for the optimal production plan of the firm.

(b) Solve for the optimal production plan of the firm if its constant marginal cost is 6.

Question 2

(a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

$$-\left[(10-a)^2 + (10-b)^2\right]$$

Suppose prices of both the goods are equal to 1.

i. Solve for the optimal consumption of the consumer when his income is 40

ii. What happens to his optimal consumption when his income goes down to 10.

(b) A monopolist faces the demand curve $q = 60 - p$ where p is measured in rupees per unit and q in thousands of units. The monopolist's total cost of production is given by $C = \frac{1}{2}q^2$

i. What is the deadweight loss due to monopoly?

ii. Suppose a government could set a price ceiling (maximum price) that the monopolist can charge. Find the price ceiling that the government should set to minimize the deadweight loss.

Question 3

(a) A cinema hall has a capacity of 150 seats. The owner can offer students a discount on the price when they show their student IDs. The demand for tickets from students is

$$D_s = 220 - 40P_s$$

where P_s is the price of tickets for students after the discount. The demand for tickets for non-students is

$$D_n = 140 - 20P_n$$

where P_n is the price of tickets for non-students.

i. What is the maximum profit the owner can make?

ii. What is the maximum profit he could make if the demand functions of students and non-students were interchanged?

(b) There are 11 traders and 6 identical (indivisible) chickens. Each trader wants to consume at most one chicken. There is also a (divisible) good called "money". Let D_i equal to 1 indicate that trader i consumes a chicken; 0 if he does not. Trader $i's$

utility function is given by $u_i D_i + m_i$, where u_i is the value he attaches to consuming a chicken, m_i is the units of money that the trader has. The valuations for the 11 traders are:

$$u_1 = 10; u_2 = 8; u_3 = 7; u_4 = 4; u_5 = 3; u_6 = 1; u_7 = u_8 = 3; u_9 = 5; u_{10} = 6; u_{11} = 8$$

Initially each trader is endowed with 25 units of money. Traders 6,7,8,9,10,11 are endowed with one chicken each.

i. What is a possible equilibrium market price (units of money per chicken in a competitive market?

ii. Is the equilibrium unique?

Question 4

(a) Consider a monopolist who faces a market demand for his product:

$$p(q) = 20 - q$$

where p is the price and q is the quantity. He has a production function given by

$$q = \min\left\{\frac{L}{2}, \frac{K}{3}\right\}$$

where L denotes labour and K denotes capital. There is a physical restriction on the availability of capital, that is, $\bar{K}$. Let both wage rates (w) and rental rates (r) be equal to 1. Find the monopoly equilibrium quantity and price when (i) when $\bar{K} = 24; (ii)\bar{K} = 18$.

(b) Define Samuelson's Weak Axiom of Revealed Preference (WARP).

(c) Prove that WARP implies non-positivity of the own-price substitution effect and the demand theorem.

Question 5

Consider two firms: 1 and 2, with their output levels denoted by q_1 and q_2. Suppose both have identical and linear cost functions, $C(q_i) = q_i$. Let the market demand function be $q = 10 - p$, where q denotes aggregate output and p the market price.

(a) Suppose the firms simultaneously decide on their output levels. Define the equilibrium in this market. Solve for the reaction functions of the two firms. Using these, find the equilibrium.

(b) Suppose the firms still compete over quantities, but both have a capacity constraint at an output level of 2. Find these reaction functions and the equilibrium in this case.

Question 6

(a) Suppose the government subsidizes housing expenditures of low income families by providing them a rupee-for-rupee subsidy for their expenditure. The Lal family qualifies for this subsidy. They spend Rs. 250 on housing, and receive Rs. 250 as subsidy from the government. Recently, a new policy has been proposed to replace the earlier policy. The new policy proposes to provide each low income family with a lump-sum transfer of Rs. 250, which can be used for housing or other goods.

i. Explain graphically if the Lal family would prefer the current program over the proposed program.
ii. Can they be indifferent between the two programs?
iii. Does the optimal consumption of housing and other goods change compared to the subsidy scheme? If yes, how?

(b) A drug company company is a monopoly supplier of Drug X which is protected by a patent. The demand for the drug is

$$p = 100 - X$$

and the monopolist's cost function is

$$C = 25 + X^2$$

i. Determine the profit maximizing price and quantity of the monopolist.
ii. Suppose the patent expires at a certain point in time, and after that any new drug company can enter the market and produce Drug X, facing the same cost function. What will be the competitive equilibrium industry output and price? How many firms will be there in the market?

Question 7

Assume that an economy's production function is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t is output at time t, K_t is the capital stock at time t and N is the fixed level of employment (number of workers), $\alpha \in (0, 1)$ is the share of output paid to capital. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where I_t is investment at time t and $\delta \in [0, 1]$ is the depreciation rate.

(a) Derive an expression for $\frac{Y}{N}$
(b) How large is the effect of an increase in the savings rate on the steady state level of output per worker.
(c) What is the savings rate that would maximize steady state consumption per worker?

Question 8

In an IS-LM model, graphically compare the effect of an expansionary monetary policy with an expansionary fiscal policy on investment (I) in (1) the short-run and (2) the medium run (where the aggregate supply and aggregate demand curves adjust). Assume that

$$I = I(i, Y)$$

where i is the interest rate and Y is the output. Also, $\frac{\partial I}{\partial i} < 0$ and $\frac{\partial I}{\partial Y} > 0$. Under which policy (expansionary monetary or fiscal), is the investment higher in the medium run?

Question 9

Suppose the economy is characterized by the following equations:

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$
$$I = b_0 + b_1 Y$$

where C is consumption, Y is the income, Y_D is the disposable income, T is tax, I is investment, and c_0, c_1, b_0, b_1 are positive constants with $c_1 < 1, b_1 < 1$. Government spending is constant.

(a) Solve for equilibrium output.

(b) What is the value of the multiplier? For the multiplier to be positive, what condition must $c_1 + b_1$ satisfy?

(c) How will equilibrium output be affected when b_0 is changed? What will happen to saving?

(d) Instead of fixed T, suppose $T = t_0 + t_1 Y$, where $t_0 > 0$ and $t_1 \in (0, 1)$. What is the effect of increase in b_0 on equilibrium Y? Is it larger or smaller than the case where taxes are autonomous?

Question 10

Consider an economy where a representative agent lives for three periods. In the first period, she is young - this is the time when she gets education. In the second period, she is middle-aged and with the level of education acquired in the first period, she generates income. More specifically, if she has h units of education in the first period, she can earn $\bar{w}h$, in the second period, where $\bar{w}$ is the exogenously given wage rate. The agent borrows funds for her education when she is young and repays with interest when she is middle aged. If in the first period, the agent borrows e, then the human capital h at the beginning of the second period becomes $h(e)$, where $\frac{dh}{de} > 0$ along with $\frac{d^2h}{de^2} < 0$ In the third period of her life, she consumes out of her savings made in the second period, that is, when she was middle aged. Assume that the exogenous rate of interest (gross) on saving or borrowing is $\bar{R}$. For simplicity, assume that an agent does not consume when she is young and, thus, the life time utility is $u\left(c^M\right) + \beta u\left(c^O\right)$, where c^M and c^O are the level of consumption when they are middle-aged and old respectively and $\beta \in (0, 1)$ is the discount factor.

(a) Write down the utility maximization problem of the agent and the first order conditions.

(b) How does the optimal level of education vary with the wage rate and the rate of interest?

ISI PEB 2015

Question 1

Consider an agent in an economy with two goods X_1 and X_2.Suppose she has income 20.Suppose also that when she consumes amounts x_1 and x_2 of the two goods respectively, she gets utility

$$u(x_1, x_2) = 2x_1 + 32x_2 - 3x_2^2$$

(a) Suppose the prices of X_1 and X_2 are each 1. What is the agent's optimal consumption bundle?

(b) Suppose the price of X_2 increases to 4, all else remaining the same. Which consumption bundle does the agent choose now?

(c) How much extra income must the agent be given to compensate her for the increase in price of X_2?

Question 2

Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $1/2$, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of $1/2$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

(a) Find the equilibrium bribe rate per connection and the social surplus.

(b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them.

(c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to $c, 0 < c < 1/2$. Find the range of values of c for which privatization increases consumers' surplus.

Question 3

Suppose the borders of a state, B, coincide with the circumference of a circle of radius $r > 0$, and its population is distributed uniformly within its borders (so that the proportion of the population living within some region of B is simply the proportion of the state's total land mass contained in that region), with total population normalized to 1.For any resident of B, the cost of travelling a distance d is kd, with $k > 0$. Every resident of B is endowed with an income of 10, and is willing to spend up to this amount to consume one unit of a good, G, which is imported from outside the state at zero transport cost. The Finance Minister of B has imposed an entry tax at the rate $100t\%$ on shipments of G brought into B. Thus, a unit of G costs $p(1+t)$ inside the borders of B, but can be purchased for just p outside; $p(1+t) < 10$. Individual residents of B have to decide whether to travel beyond its borders to consume the good or to purchase it inside the state. Individuals can travel anywhere to shop and consume, but have to return to their place of origin afterwards.

(a) Find the proportion of the population of B which will evade the entry tax by shopping outside the state.

(b) Find the social welfare-maximizing tax rate. Also find the necessary and sufficient conditions for it to be identical to the revenue-maximizing tax rate.

(c) Assume that the revenue-maximizing tax rate is initially positive. Find the elasticity of tax revenue with respect to the external price of G, supposing that the Finance Minister always chooses the revenue-maximizing tax rate.

Question 4

Suppose there are two firms, 1 and 2 , each producing chocolate, at 0 marginal cost. However, one firm's product is not identical to the product of the other. The inverse demand functions are as follows:

$$p_1 = A_1 - b_{11}q_1 - b_{12}q_2, p_2 = A_2 - b_{21}q_1 - b_{22}q_2$$

where p_1 and q_1 are respectively price obtained and quantity produced by firm 1 and 2 and p_2 and q_2 are respectively price obtained and quantity produced by firm 2 . $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are all positive.Assume the firms choose independently how much to produce.

(a) How much do the two firms produce, assuming both produce positive quantities?

(b) What conditions on the parameters $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are together both necessary and sufficient to ensure that both firms produce positive quantities?

(c) Under what set of conditions on these parameters does this model reduce to the standard Cournot model?

Question 5

Suppose a firm manufactures a good with labor as the only input. Its production function is $Q = L$, where Q is output and L is total labor input employed. Suppose further that the firm is a monopolist in the product market and a monopsonist in the labor market. Workers may be male (M) or female (F); thus, $L = L_M + L_F$.Let the inverse demand function for output and the supply functions for gender-specific labor be respectively $p = A - \frac{Q}{2}; L_i = w_i^{\varepsilon_i}, \varepsilon_i > 0$; where p is the price received per unit of the good and w_i is the wage the firm pays to each unit of labor of gender $i, i \in \{M, F\}$.Let $\varepsilon_M \varepsilon_F = 1$. Suppose, in equilibrium, the firm is observed to hire both M and F workers, but pay M workers double the wage rate that it pays F workers.

(a) Derive the exact numerical value of the elasticity of supply of male labor.

(b) What happens to total male labor income as a proportion of total labor income when the output demand parameter A increases? Prove your claim.

Question 6

An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good(Y) by means of a production function $Y = F(L); F' > 0, F'' < 0$, and maximizes profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate.The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by:

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\frac{M}{P} - d(L)$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour; with $d' > 0, d'' > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \bar{M} - PT$$

where $\bar{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G.The government's budget constraint is given by:

$$M - \bar{M} = PG - PT$$

Goods market clearing implies: $Y = C + G$

(a) Prove that $\frac{dY}{dG} \in (0, 1)$, and that government expenditure crowds out private consumption (i.e.,, $\frac{dC}{dG} < 0$).

(b) Show that everything else remaining the same, a rise in $\bar{M}$ leads to an equiproportionate rise in P.

Question 7

Consider the Solow growth model in continuous time, where the exogenous rate of technological progress, g, is zero. Consider an intensive form production function given by:

$$f(k) = k^4 - 6k^3 + 11k^2 - 6k \quad (1)$$

where $k = \frac{K}{L}$ (the capital labour ratio).

(a) Specify the assumptions made with regard to the underlying extensive form production function $F(K, L)$ in the Solow growth model, and explain which ones among these assumptions are violated by (1).

(b) Graphically show that, with a suitable value of $(n + \delta)$, where n is the population growth rate, and $\delta \in [0, 1]$ is the depreciation rate on capital, there exist three steady state equilibria.

(c) Explain which of these steady state equilibria are locally unstable, and which are locally stable. Also explain whether any of these equilibria can be globally stable.

Question 8

Consider a standard Solow model in discrete time, with the law of motion of capital is given by

$$K(t+1) = (1-\delta)K(t) + I(t)$$

where $I(t)$ is investment at time t and $K(t)$ is the capital stock at time t; the capital stock depreciates at the rate $\delta \in [0,1]$.Suppose output, $Y(t)$, is augmented by government spending, $G(t)$, in every period, and that the economy is closed; thus:

$$Y(t) = C(t) + I(t) + G(t)$$

where $C(t)$ is consumption at time t. Imagine that government spending is given by:

$$G(t) = \sigma Y(t)$$

where $\sigma \in [0,1]$

(a) Suppose that: $C(t) = (\emptyset - \lambda\sigma)Y(t)$; where $\lambda \in [0,1]$. Derive the effect of higher government spending (in the form of higher σ) on the steady state equilibrium.

(b) Does a higher σ lead to a lower value of the capital stock in every period (i.e, along the entire transition path)? Prove your claim.

ISI PEB 2016

Question 1

Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

(a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

(b) What is the competitive equilibrium in this economy? Justify your answer.

(c) Are the perfectly competitive equilibria Pareto optimal?

(d) Now consider another economy where everything is as before, apart from individual 2 's preferences, which are as follows: (a) among any two any bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy.

Question 2

Consider a monopolist who can sell in the domestic market, as well as in the export market. In the domestic market she faces a demand $p_d = 10 - q_d$, where p_d and q_d are domestic price and demand respectively. In the export market she can sell unlimited quantities at a price of 4. Suppose the monopolist has a single plant with cost function $\frac{q^2}{4}$.

(a) Solve for total output, domestic sale and exports of the monopolist.

(b) Solve for the domestic and world welfare at this equilibrium.

Question 3

A consumer consumes electricity, denoted by E, and butter, denoted by B. The per unit price of B is 1. To consume electricity the consumer has to pay a fixed charge R, and a per unit price of p. If consumption of $E \leq \frac{1}{2}$ then $p = 1$; otherwise $p = 2$. The utility function of the consumer is $3E + B$, and her income is $I > R$

(a) Draw the consumer's budget line.

(b) If $R = 0$ and $I = 1$, find the consumer's optimal consumption of E and B.

(c) Consider a different pricing scheme where there is a rental charge of R and the price of E is 1 for any $X \leq 1/2$, and every additional unit beyond $\frac{1}{2}$ is priced at $p = 2$. Find the optimal consumption of B and E when $R = 1$ and $I = 3$

Question 4

A monopoly publishing house publishes a magazine, earning revenue from selling the magazine, as well as by publishing advertisements. Thus $R = q \cdot p(q) + A(q)$, where R is total revenue, q denotes quantity, $p(q)$ is the inverse demand function, and $A(q)$ is the advertising revenue. Assume that $p(q)$ is decreasing and $A(q)$ is increasing in q. The cost of production $c(q)$ is also increasing in the quantity sold. Assume all functions are twice differentiable in q.

(a) Derive the profit-maximising outcome.

(b) Is the marginal revenue curve necessarily negatively sloped?

(c) Can the monopolist fix the price of the magazine below the marginal cost of production?

Question 5

Consider a Solow style growth model where the production function is given by

$$Y_t = A_t F(K_t, H_t)$$

where Y_t = output of the final good, K_t is the capital stock, A_t = the level of technology, and H_t = the quantity of labor used in production (the labor force). Assume technology is equal to $A_t = A_0(1+\alpha)^t$ where $\alpha > 0$ is the growth rate of technology, A_0 is the time 0 level of technology, and $H_{t+1} = (1+n)H_t$, where $n > 0$ is the labour force growth rate. The production function is homogeneous of degree 1 and satisfies the usual properties. (Assume that inputs are essential and Inada conditions hold). Assume that capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is the level of investment.

(a) Define $y_t = \frac{Y_t}{H_t}$. Show that

$$y_t = A_t f(k_t)$$

where $f(k) = F(k, 1)$

(b) Define $k_t = \frac{K_t}{H_t}$ and $i_t = \frac{I_t}{H_t}$. Show that

$$k_{t+1} = \frac{(1-\delta)k_t + i_t}{1+n}$$

(c) Suppose the savings rate is given by $s_t = \sigma y_t$ where $\sigma \in [0, 1]$. Derive the condition that determines the steady state capital stock when $\alpha = 0$. How many non-zero steady states are there?

(d) Let $\gamma_t = \frac{k_{t+1}}{k_t}$ be the gross growth rate. Suppose $\alpha = 0$. Derive an expression for γ_t and evaluate and discuss the sign for $\frac{d\gamma_t}{dk_t}$

(e) Let $f(k_t) = k_t^{\theta}, A_0 = 1$, and $\alpha > 0$. Along a balanced growth path show that $\frac{k_{t+1}}{k_t}$ and $\frac{y_{c+1}}{y_t}$ grow at the same rate.

Question 6

Consider the aggregate supply curve for an economy given by

$$P_t = P_t^e(1+\mu)F(u_t, z)$$

where P_t = actual price level at time period t, P_t^e = expected prices at time t, and the function, F, given by,

$$F(u_t, z) = 1 - \alpha u_t + z$$

captures the effects of the unemployment rate (u_t) at time t and the level of unemployment benefits (z) on the price level (through their effects on wages). Assume $\mu > 0$ denotes the monopoly markup. Assume μ and z are constant.

(a) Show that the aggregate supply curve can be transformed to be written in terms of π_t (the inflation rate) and the expected inflation rate, π_t^e, i.e. $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$

(b) Now assume that $\pi_t^e = \theta \pi_{t-1}$ where $\theta > 0$. What is this equation called ? Re-write the equation in the above bullet and interpret when $\theta = 1$ and $\theta \neq 1$.

(c) Let $\pi_t^e = \pi_{t-1}$ Derive the natural rate of unemployment, and express the change in the inflation rate in terms of the natural rate. Briefly interpret this equation.

(d) How would you think about wage indexation in this model ? Does wage indexation increase the effect of unemployment on inflation? Assume $\pi_t^e = \pi_{t-1}$.

Question 7

Consider an inter-temporal choice problem in which a consumer maximises utility,

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\delta}$$

where c_i is the consumption in period i, $i = 1, 2$, and δ is the discount factor (measure of the consumer's impatience), subject to

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \equiv W$$

where Y_i is the consumer's income in period $i = 1, 2$, and r is the rate of interest. Assume $c_i > 1 \forall i$

(a) Let $u(c_i) = \log(c_i)$. Find a condition such that there is consumption smoothing.

(b) Plot the two cases where (a) the consumer biases its consumption towards the future, and (b) where the consumer biases it consumption towards the present. Put c_2 on the vertical axis and c_1 on the horizontal axis.

(c) Suppose there is consumption smoothing. Solve for $c_1^* = c_1(r, Y_1, Y_2)$. Interpret this equation.

(d) Define Y_P, the permanent income, as that constant stream of income (Y_P, Y_P) which gives the same lifetime income as does the fluctuating income stream (Y_1, Y_2). What does this imply about the optimal choice of c_1, c_2, and Y_P? Interpret your result graphically.

Question 8

Consider a cake of size 1 which can be divided between two individuals, A and B. Let α(resp. β) be the amount allocated to A(resp. B), where $\alpha + \beta = 1$ and $0 \leq \alpha, \beta \leq 1$ Agents A's utility function is $u_A(\alpha) = \alpha$ and that of agent B is $u_B(\beta) = \beta$

(a) What is the set of Pareto optimal allocations in this economy?

(b) Suppose A is asked to cut the cake in two parts, after which B can choose which of the two segments to pick for herself, leaving the other segment for agent A. How should A cut the cake?

(c) Suppose A is altruistic, and his utility function puts weight on what B obtains, i.e. $u_A(\alpha, \beta) = \alpha + \mu\beta$, where μ is the weight on agent B 's utility. (a) If $0 < \mu < 1$, does the answer to either 8(i) or 8 (ii) change? (b) What if $\mu > 1$?

Question 9

A firm uses four inputs to produce an output with a production function

$$f(x_1, x_2, x_3, x_4) = \min\{x_1, x_2\} + \min\{x_3, x_4\}$$

(a) Suppose that 1 unit of output is to be produced and factor prices are 1,2,3 and 4 for x_1, x_2, x_3 and x_4 respectively. Solve for the optimal factor demands.

(b) Derive the cost function.

(c) What kind of returns to scale does this technology exhibit?

Question 10

Consider an IS-LM model where the sectoral demand functions are given by

$$C = 90 + 0.75Y$$
$$G = 30, I = 300 - 50r$$
$$\left(\frac{M}{P}\right)_d = 0.25Y - 62.5r, \left(\frac{M}{P}\right)_s = 500$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Now consider an initial situation where $Y = 2500, r = 1/5$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 2, Y = 2500)$?

ISI PEB 2017

Question 1

A researcher has 100 hours of work which have to be allocated between two research assistants, Aditya and Gaurav. If Aditya is allocated x hours of work, his utility is $-(x-20)^2$. If Gaurav is allocated x hours of work, his utility is $-(x-30)^2$. The researcher is considering two proposals: [I] Aditya works for 60 hours and Gaurav works for 40 hours. [II] Aditya works for 90 hours and Gaurav works for 10 hours. Which of the following statements is correct.

A. Proposal I is Pareto efficient but Proposal II is not.

B. Proposal II is Pareto efficient but Proposal I is not.

C. Both proposals are Pareto efficient.

D. Neither proposal is Pareto efficient.

Question 2

The industry demand curve for tea is: $Q = 1800 - 200P$. The industry exhibits constant long run average cost (ATC) at all levels of output of Rs. 1.50 per unit of output. Which market form(s) - perfect competition, pure monopoly, and first-degree price discrimination - has the highest total market (that is, producer + consumer) surplus?

A. perfect competition

B. pure monopoly

C. first degree price discrimination

D. perfect competition and first degree price discrimination

Question 3

The following information will be used in the next question also. OIL Inc. is a monopoly in the local oil refinement market. The demand for refined oil is

$$Q = 75 - P$$

where P is the price in Rupees and Q is the quantity, while the marginal cost of production is

$$\text{MC} = 0.5Q$$

The fixed cost is zero. Pollution is emitted in the refinement of oil which generates a marginal external cost (MEC) equal to Rs. 31 per unit. What is the level of Q that maximizes social surplus?

A. 50

B. $29\frac{1}{3}$

C. 17.6

D. 44

Question 4

Refer to the previous question. Suppose the government decides to impose a per unit pollution fee on OIL Inc. At what level should the fee (in Rs./unit) be set to produce the level of output that maximizes social surplus? You may use the fact that the marginal revenue is given by: MR = $75 - 2Q$.

A. 1/3

B. 2

C. 3/4

D. 5/3

Question 5

Mr. X has an exogenous income W, and his utility function from consumption is given by $U(c)$. With probability p, an accident can occur. If it occurs, the monetary equivalent of the damage is T. Mr. X can however affect the accident probability p by taking prevention effort e. In particular e can take two values: 0 and $a > 0$. Assume that $p(0) > p(a)$. Let us also assume that the utility cost of effort is Ae^2. Calculate the value of A below which effort will be undertaken.

A. $\dfrac{[p(a) - p(0)][U(W - T) - U(W)]}{a^2}$

B. $\dfrac{p(a) - p(0)}{U(W - T) - U(W)}$

C. $\dfrac{p(a)p(0)a^2}{U(W - T) - U(W)}$

D. $\dfrac{p(a)/p(0)}{U(W - T)/U(W)}a^2$

Question 6

Suppose Mr. X maximizes inter-temporal utility for 2 periods. His total utility is given by

$$\log(c_1) + \beta \log(c_2)$$

where $\beta \in (0, 1)$ and c_1 and c_2 are his consumption in period 1 and period 2, respectively. Suppose he earns a wage only in period 1 and it is given by W. He saves for the second period on which he enjoys a gross return of $(1 + r)$ where $r > 0$ is the net interest rate. Suppose the government implements a scheme where $T \geq 0$ is collected from agents (this also from Mr. X) in the first period, and gives the same amount T back in the second period. What is the optimum T for which his total utility is maximized?

A. $T = 0$

B. $T = \dfrac{W}{2\beta}$

C. $T = \dfrac{\beta W}{2(1 - \beta)}$

D. $T = \dfrac{W}{2(1 - \beta)}$

Question 7

Suppose there is one company in an economy which has a fixed supply of shares in the short run. Suppose there is new information that causes expectations of lower profits. How does this new stock market equilibrium affect final output and the final price level of the economy if you assume that autonomous consumption spending and household wealth are positively related?

A. real GDP increases; price decreases

B. real GDP decreases, price increases

C. real GDP decreases, price decreases

D. real GDP increases, price stays constant

Question 8

A monopolist faces a demand function, $p = 10 - q$. It has two plants at its disposal. The cost of producing q_1 in the first plant is $300 + q_1^2$ if $q_1 > 0$, and 0 otherwise. The cost of producing q_2 in the second plant is $200 + q_2^2$ if $q_2 > 0$, and 0 otherwise. What are the optimal production levels in two plants?

A. 10 units in both plants

B. 20 units in the first plant and 10 in the second.

C. 0 units in the first plant and 15 in the second

D. None of the above

Question 9

Consider a firm facing three consumers, 1, 2, and 3, with the following valuations for two goods X and Y (All consumers consume at most 1 unit of X and 1 unit of Y)

Consumer	X	Y
1	7	1
2	4	5
3	1	6

The firm can produce both the goods at a cost of zero. Suppose the firm can supply both goods at a constant per unit price of p_X for X, and p_Y for Y. It can also supply the two goods as a bundle, for a price of p_{XY}. The optimal vector of prices (p_X, p_Y, p_{XY}) is given by

A. (7,6,9)

B. (4,1,4)

C. (7,7,7)

D. None of the above

Question 10

Two individuals, Bishal (B) and Julie (J), discover a stream of mountain spring water. They each separately decide to bottle some of this water and sell it. For simplicity, presume that the cost of production is zero. The market demand for bottled water is given by $P = 90 - 0.25Q$, where P is price per bottle and Q is the number of bottles. What would Bishal's output Q_B, Julie's output Q_J, and the market price be if the two individuals behaved as Cournot duopolists?

A. $Q_B = 120; Q_J = 120; P = 42$

B. $Q_B = 90; Q_J = 90; P = 30$

C. $Q_B = 120; Q_J = 120; P = 30$

D. $Q_B = 100; Q_J = 120; P = 30$

Question 11

The next three questions (11,12,13) are to be answered based on the following information: Consider the following model of a closed economy:

$$\begin{aligned}
\Delta Y &= \Delta C + \Delta I + \Delta G \\
\Delta C &= c\Delta Y_d \\
\Delta Y_d &= \Delta Y - \Delta T \\
\Delta T &= t\Delta Y + \Delta T_0
\end{aligned}$$

where ΔY = change in GDP, ΔC = change in consumption, ΔI = change in private investment, ΔG = change in government spending, ΔY_d = change in disposable income (i.e. after tax income), ΔT = the change in total tax collections, $t \in (0, 1)$ is the tax rate, and ΔT_0 = the change in that portion of tax collections that can be altered by government fiscal policy measures. The value of the balanced budget multiplier (in terms of G and T_0) is given by:

A. $\frac{1}{1-c(1-t)}$

B. $\frac{-c}{1-c(1-t)}$

C. $\frac{1-c}{1-c(1-t)}$

D. none of the above

Question 12

Refer to the previous question. Suppose the marginal to consumer, $c = 0.8$ and $t = 0.375$. The value of the government expenditure multiplier is

A. 2

B. -1.6

C. 0.4

D. 0.5

Question 13

Refer to the previous question. Suppose the marginal to consumer, $c = 0.8$ and $t = 0.375$. The value of the tax multiplier (with respect to T_0) is

A. -1.6

B. 2

C. 0.4

D. 0.3

Question 14

In the IS-LM model, a policy plan to increase national savings (public and private) without changing the level of GDP, using any combination of fiscal and monetary policy involves

A. contractionary fiscal policy, contractionary monetary policy

B. expansionary fiscal policy, contractionary monetary policy

C. contractionary fiscal policy, expansionary monetary policy

D. expansionary fiscal policy, expansionary monetary policy

Question 15

Consider the IS-LM-BP model with flexible exchange rates but with no capital mobility. Consider an increase in the money supply. At the new equilibrium, the interest rate is . . ., the exchange rate is . . ., and the level of GDP is . . ., respectively.

A. higher, lower, higher

B. lower, higher, higher

C. lower, higher, lower

D. higher, lower, lower

Question 16

Consider a Solow model of an economy that is characterized by the following parameters: population growth, n; the depreciation rate, δ; the level of technology, A; and the share of capital in output, α. Per-capita consumption is given by $c = (1-s)y$ where s is the exogenous savings rate, and $y = Ak^\alpha$, where y denotes output per-capita, and k denotes the per-capita capital stock. The economy's golden rule capital stock is determined by which of the following conditions?

A. $\frac{\partial c}{\partial k} = Ak^\alpha - (n+\delta)k = 0$

B. $\frac{\partial c}{\partial k} = \alpha Ak^{\alpha-1} - (n+\delta) = 0$

C. $\frac{\partial c}{\partial k} = (n+\delta)k - sAk^\alpha = 0$

D. none of the above

Question 17

In the Ramsey model, also known as the optimal growth model, with population growth n and an exogenous rate of growth of technological progress g, the steady state growth rates of aggregate output Y, aggregate capital K and aggregate consumption C are

A. $0, 0, 0$

B. $n+g, n+g, n+g$

C. $g, n+g, n$

D. $n+g, n+g, g$

Question 18

Consider the standard formulation of the Philips Curve,

$$\pi_t - \pi_t^e = -\alpha(u_t - u_n)$$

where π_t is the current inflation rate, π_t^e is the expected inflation rate, α is a parameter, u_n is the natural rate of unemployment. Suppose the economy has two types of labour contracts: a proportion, λ, that are indexed to actual inflation, π_t, and a proportion, $1-\lambda$, that are not indexed and simply respond to last year's inflation, π_{t-1}. Wage indexation (relative to no indexation) will ... the effect of unemployment on inflation.

A. strongly decrease

B. increase

C. not change

D. mildly decrease

Question 19

Consider a Harrod-Domar style growth model with a (i) Leontief aggregate production function, (ii) no technological progress, and (iii) constant savings rate. Let K and L denote the level of capital and labor employed in the economy. Output Y is produced according to $Y = \min(AK, BL)$ where A and B are positive constants. Let $\overline{L}$ be the full employment level. Under what condition will there be positive unemployment.

A. $AK > B\overline{L}$

B. $AK < B\overline{L}$

C. $AK = B\overline{L}$

D. none of the above

Question 20

The next two questions (20 and 21) are to be answered together. People in a certain city get utility from driving their cars but each car releases k units of pollution per km driven. The net utility of each person is his or her utility from driving , v, minus the total pollution generated by everyone else. Person i's net utility is given by: $U_i(x_1, \ldots, x_n) = v(x_i) - \sum_{j \neq i \;\&\; j=1}^{n} kx_j$ where x_j is km driven by person j, n is the city population, and the utility of driving v has an inverted U-shape with $v(0) = 0, \lim_{x \to 0^+} v'(x) = \infty, v''(x) < 0$, and $v(\overline{x}) = 0$ for some $\overline{x} > 0$. In an unregulated city, an increase in population will

A. increase the km driven per person

B. decrease the km driven per person

C. leave the km driven per person unchanged

D. may or may not increase the km driven per person

Question 21

Refer to the information given in the previous question. A city planner decides to impose a tax per km driven and sets the tax rate in order to maximize the total net utility of the residents. Then, if the population increases, the optimal tax will

A. increase

B. decrease

C. stay unchanged

D. may or may not increase

Question 22

The production function: $F(L, K) = (L + 10)^{\frac{1}{2}} K^{\frac{1}{2}}$ has

A. increasing returns to scale

B. constant returns to scale

C. decreasing returns to scale

D. none of the above

Question 23

Consider the production functions: $F(L, K) = L^{\frac{1}{2}} K^{\frac{2}{3}}$ and $G(L, K) = LK$ where L denotes labour and K denotes capital.

A. F is consistent with the law of diminishing returns to capital but G is not

B. G is consistent with the law of diminishing returns to capital but F is not

C. Both F and G are consistent with the law of diminishing returns to capital

D. Neither F nor G is consistent with the law of diminishing returns to capital

Question 24

A public good is one that is non-rivalrous and non-excludable. Consider a cable TV channel and a congested city street.

A. A cable TV channel is a public good but a congested city street is not

B. A congested city street is a public good but a cable TV channel is not

C. Neither is a public good

D. Both are public goods

Question 25

Firm A's cost of producing output level $y > 0$ is $c_A(y) = 1 + y$ while firm B's cost of producing of producing output level y is $c_B(y) = y(1 - y)^2$.

A. A can operate in a perfectly competitive industry but B cannot

B. B can operate in a perfectly competitive industry but A cannot

C. Neither could operate in a perfectly competitive industry

D. Either could operate in a perfectly competitive industry

Question 26

Suppose we generally refer to a New Keynesian model as a model with a non vertical aggregate supply (AS) curve. Under sticky prices, the AS curve will be ..., and under sticky wages, the AS curve will be ..., respectively.

A. horizontal, upward sloping

B. upward sloping, upward sloping

C. downward sloping, horizontal

D. upward sloping, horizontal

Question 27

With perfect capital mobility, and ..., monetary policy is ... at influencing output.

A. fixed exchange rates, effective

B. fixed exchange rates, ineffective

C. flexible exchange rates, ineffective

D. none of the above are correct

Question 28

The next three questions (28,29 and 30) use the following information. Consider an economy with two goods x and y, and two consumers, A and B, with endowments (x, y) given by $(1, 0)$ and $(0, 1)$ respectively. A's utility is $U_A(x, y) = x + 2y$ while B's utility is $U_B(x, y) = 2x + y$. Using an Edgeworth box with x measured on the horizontal axis and y measured on the vertical axis, with A's origin in the bottom-left corner and B's origin in the top-right corner, the set of Pareto-optimal allocations is

A. a straight line segment

B. the bottom and right edges of the box

C. the left and top edges of the box

D. none of the above

Question 29

Referring to the information given in the previous question, the following allocations are the ones that may be achieved in some competitive equilibrium.

A. $(0, 1)$

B. The line segment joining $(0, 1/2)$ to $(0, 1)$ and the line segment joining $(0, 1)$ to $(1/2, 1)$

C. The line segment joining $(1/2, 0)$ to $(1, 0)$ and the line segment joining $(1, 0)$ to $(1, 1/2)$

D. $(1, 0)$

Question 30

Referring to the information given in the previous two questions, if the price of y is 1, then the price of x in a competitive equilibrium

A. must be 1/2

B. must be 1

C. must be 2

D. could be any of the above

ISI PEB 2018

Question 1

Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is $\frac{1}{2}$, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of $\frac{1}{2}$ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

(a) Find the equilibrium bribe rate per connection and the social surplus.

(b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them.

(c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to $c, 0 < c < \frac{1}{2}$. Find the range of values of c for which privatization increases consumers' surplus.

Question 2

Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $\min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

(a) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

(b) What is the competitive equilibrium in this economy? Justify your answer.

(c) Are the perfectly competitive equilibria Pareto optimal?

(d) Now consider another economy where everything is as before, apart from individual 2 's preferences, which are as follows: (a) among any two bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy.

Question 3

An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm. The firm produces a single good (Y) by means of a production function $Y = F(L), F'(L) > 0, F''(L) < 0$, and maximizes profits $\Pi = PY - WL$, where P is the price of Y and W is the wage rate.

The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\left(\frac{M}{P}\right) - d(L)$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour with $d'(L) > 0, d''(L) > 0$. The household's budget constraint is given by:

$$PC + M = WL + \Pi + \bar{M} - PT$$

where $\bar{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G. The government's budget constraint is given by:

$$M - \bar{M} = PG - PT$$

Goods market clearing implies $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0, 1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$).

(b) Show that everything else remaining the same, a rise in $\bar{M}$ leads to an equiproportionate rise in P.

Question 4

Consider an IS-LM model where the sectoral demand functions are given by

$$\begin{aligned} C &= 90 + 0.75Y \\ G &= 30 \\ I &= 300 - 50r \\ \left(\frac{M}{P}\right)_d &= 0.25Y - 62.5r \\ \left(\frac{M}{P}\right)_s &= 500 \end{aligned}$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Consider an initial situation where $Y = 2500, r = \frac{1}{5}$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 0.2, Y = 2500)$?

Question 5

Answer the following questions.

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \frac{|x|}{2x} \forall x \in \mathbb{R} \backslash \{0\}$$

Can $f(0)$ be defined in a way such that f is continuous at 0? Justify your answer.

(b) Consider the following optimization problem:

$$\max_{x \in [0,\beta|} x(1-x)$$

where $\beta \in [0,1]$. Let x^* be an optimal solution of the above optimization problem. For what values of β will we have $x^* = \beta$?

(c) A firm is producing two products a and b. The market price (per unit) of a and b are respectively 3 and 2 . The firm has resources to produce only 10 units of a and b together. Also, the quantity of a produced cannot exceed double the quantity of b produced. What is the revenue-maximizing production plan (i.e., how many units of a and b) of the firm?

Question 6

Answer the following questions.

(a) A slip of paper is given to person A, who marks it with either (+) or (−). The probability of her writing (+) is $\frac{1}{3}$. Then, the slip is passed sequentially to B, C, and D. Each of them either changes the sign on the slip with probability $\frac{2}{3}$ or leaves it as it is with probability $\frac{1}{3}$.

i. Compute the probability that the final sign is (+) if A wrote (+).

ii. Compute the probability that the final sign is (+) if A wrote (−).

iii. Compute the probability that A wrote (+) if the final sign is (+).

(b) There are n houses on a street numbered $h_1, \ldots, h_n$. Each house can either be painted BLUE or RED.

i. How many ways can the houses $h_1, \ldots, h_n$ be painted?

ii. Suppose $n \geq 4$ and the houses are situated on n points on a circle. There is an additional constraint on painting the houses: exactly two houses need to be painted BLUE and they cannot be next to each other. How many ways can the houses $h_1, \ldots, h_n$ be painted under this new constraint?

iii. How will your answer to the previous question change if the house are located on n points on a line.

ISI PEB 2019

Question 1

Ms. A earns Rs. 25,000 in period 1 and Rs. 15,000 in period 2. Mr. B earns Rs. 15,000 in period 1 and Rs. 30,000 in period 2 . They can borrow money at an interest rate of 200%, and can lend money at a rate of 0%. They like both consumption in period 1 (C_1) and consumption in period 2 (C_2) , and their preferences are such that their chosen consumption bundles will always lie on their budget lines.

(a) Write down the equations of their budget constraints and draw their budget lines in the same figure by plotting consumption in period 1 (C_1) (in thousand rupees) on x -axis and consumption in period 2 (C_2) (in thousand rupees) on y -axis.

(b) Given the income profile and the market interest rates, Mr. B chooses to borrow Rs. 5,000 in period 1. Give an example of a consumption profile (that is, (C_1, C_2)) such that, if Ms. A chooses this profile, we would know for sure that Ms. A and Mr. B have different preferences for consumption in period 1 (C_1) and consumption in period 2 (C_2) . Give a clear explanation for your answer.

(c) Suppose now that Ms. A and Mr. B have the same preferences for C_1 and C_2, and, as in part (b), Mr.B borrows Rs. 5,000 in period 1

i. Suppose that Ms. A chooses to be a lender in period one. Find out, with a clear explanation, the maximum amount that she will lend in period 1 consistent with the fact that they have the same preferences for C_1 and C_2

ii. Explain clearly whether Mr.B is better off than Ms. A.

Question 2

Consider an exchange economy with two agents 1 and 2 and two goods X and Y. There is one unit of both goods in the economy. An allocation is a pair $\{(X_1, Y_1), (X_2, Y_2)\}$ where $X_1 + X_2 = 1, Y_1 + Y_2 = 1$, and (X_1, Y_1) and (X_2, Y_2) are the consumption bundles of agents 1 and 2 respectively. The utility function for agent 1 is given by $u_1(X_1, Y_1) = X_1 \cdot Y_1$ and that of agent 2 by $u_2(X_2, Y_2) = 2X_2 + Y_2$

(a) Describe the set of Pareto-efficient allocations in the economy.

(b) An allocation $\{(X_1, Y_1), (X_2, Y_2)\}$ is envy-free if no agent strictly prefers the consumption bundle of the other agent to her own, that is, $u_1(X_1, Y_1) \geq u_1(X_2, Y_2)$ and $u_2(X_2, Y_2) \geq u_2(X_1, Y_1)$

i. Consider each of the two statements below. Decide whether they are true or false. Justify your answer with a proof or a counter-example as appropriate.

α) All Pareto-efficient allocations are envy-free.

β) All envy-free allocations are Pareto-efficient.

ii. Describe the set of envy-free allocations in the economy.

(c) Suppose each agent has an endowment of half-unit of each good. Prove without direct computation that the competitive equilibrium allocation is both Pareto-efficient and envy-free.

Question 3

Two flat-mates, 1 and 2, rent a flat and play their own music on the only CD player owned by the flat-owner. They both like their own music, but dislike the music played by the other person. Given the timing constraints, each one must play her own music when the other person is also present. Let m_i denote the amount of music played by i, and Y_i denote her amount of money holding. Individual i 's utility function is

$$u_i\left(m_1, m_2, Y_i\right)=8 m_i-2 m_i^2-\frac{3}{2} m_j^2+Y_i, \quad i, j=1,2, i \neq j$$

(a) How much music would each individual play? What is the efficient amount of music for each individual? Is the amount of music actually played more or less than the efficient level? Explain the economic intuition for your answer.

(b) Suppose that individual 2 is considering to gift a headphone to her flat-mate on her birthday. Assume that she does not get any utility from just gift-giving. What is the maximum price she is willing to pay for the headphone?

(c) Suppose that the price of the headphone is Rs. 11. Does it make sense for the two flat-mates to jointly buy a headphone, sharing the price equally, and making a binding commitment that they would each listen to their own music only via the headphone?

(d) Now suppose that the CD player is owned by individual 1 so that she can prevent individual 2 from playing any music at all. Suppose individual 1 can offer a take-it-or-leave-it contract that looks like the following: "I shall play music at a level $\bar{m}_1$, and you can play music at the level $\bar{m}_2$ in return for a sum of Rs. T." In case the offered contract is rejected, individual 1 selects m_1 unilaterally, and individual 2 cannot play any music of her choice. Solve for the optimal levels of $\bar{m}_1, \bar{m}_2$ and T Discuss the economic intuition for your answer.

Question 4

Consider a country where there are only two provinces $-A$ and B. The production function to produce a single output Y is given by $Y=F\left(N^A+N^B\right)$ where F is a concave function and N^i represents employees from province $i, i=A, B$. Wages paid to the employees are given by $W^i, i=A, B$. Price of the final good Y is denoted by P. The employers are price takers and take P, W^A and W^B as given.

(a) Write down the expression for an employer's profit as a function of N^A and N^B, $\pi\left(N^A, N^B\right)$

(b) An employer chooses N^A and N^B to maximize

$$u\left(N^A, N^B\right)=u\left(\pi\left(N^A, N^B\right), N^A, N^B\right)$$

where $\frac{\partial u}{\partial \pi}>0, \frac{\partial u}{\partial N^A}>0$ and $\frac{\partial u}{\partial N^B}<0$. The last two conditions on $u\left(N^A, N^B\right)$ imply that the employer prefers employees from province A but dislikes employees from province B Write down the first order conditions for the employer's maximization problem assuming an interior solution.

(c) In equilibrium do the employees from different provinces get the same wage? If yes, explain your answer. If not, then determine, with a clear explanation, which employees are paid more and by how much.

Question 5

Consider a concave utility function $u(c, l)$ where c represents consumption good and l represents labour supply (working hours, to be precise). While utility increases with the level of consumption good, increasing working hours reduces utility. Wage per hour of labour is given by w, thus working for l hours will ensure wl amount of total wage which is denoted by y, that is, $y = wl$. Given this, the utility function can be written as $u\left(c, \frac{y}{w}\right)$. The price of the consumption good c is given by p. Also $\bar{L}$ is a fixed number of hours representing total time available to an agent and $\bar{L} - l$ represents leisure. [In all the figures you are asked to draw below, plot y on x -axis and c on y -axis.]

(a) Derive the slope of an indifference curve for the utility function $u\left(c, \frac{y}{w}\right)$ on the $y - c$ plane.

(b) Demonstrate the agent's utility maximizing choice of y and c in a figure by plotting her budget line and indifference curves for the utility function $u\left(c, \frac{y}{w}\right)$

(c) Experiment 1: Suppose there is an increase in w. Demon strate the agent's new utility maximizing choice of y and c in the same figure as in part b. [Show clearly how the agent's budget line and/or indifference curves change as a result of the increase in w.] Compare the old and new choices with a brief economic explanation.

(d) Experiment 2 : Suppose, instead of an increase in w, there is a tax imposed on income. That is, the after-tax income of the agent is $(1 - \tau)y$ where τ is the proportional tax rate. In a new figure demonstrate the agent's new as well as old (as in part b) utility maximizing choices of y and c. [Show clearly how the agent's budget line and/or indifference curves change as a result of this proportional tax.] Compare the old and new choices with a brief economic explanation.

Question 6

Consider an agent who lives for three periods but consumes only in periods two and three where the consumptions are denoted by c_2 and c_3 respectively. Her utility is given by $u(c_2, c_3) = \log(c_2) + \beta \log(c_3)$, where $0 < \beta < 1$ is the discount factor reflecting her time preference. She invests an amount e in education in the first period which she borrows from the market at a given interest rate $r > 0$. Her income in the second period is $w \cdot h(e)$ where w is a fixed wage rate per unit of human capital and $h(e)$ is the amount of human capital that results from investment in education (e) in the first period. Assume that $h(e)$ is an increasing and concave function of e. The agent repays her education loan in the second period. She has no income in the third period. But she can save (s) in the second period from her income on which she receives the return $s(1 + r)$ in the third period to meet her consumption expenditure.

(a) Write down the agent's period 2 and period 3 budget constraints separately.

(b) Set up the agent's utility maximization problem by showing her choice variables clearly.

(c) Write down the first order conditions for the agent's utility maximization problem.

(d) Derive the ratio of consumptions in period 2 and period 3 ,

(e) Explain how investment in education, e, depends on the preference parameter β

Question 7

Consider a street represented by the interval $[0,1]$. Three agents, $\{1,2,3\}$, live on this street. Agent $i \in \{1,2,3\}$ lives at $x_i \in [0,1]$ and assume that $x_1 \leq x_2 \leq x_3$. Suppose we locate a hospital at a point $p \in [0,1]$

(a) We say p is square-optimal if it minimizes $\sum_{i=1}^{3}\left(x_i - p\right)^2$ Derive the square optimal value of p

(b) We say p is absolute-optimal if it minimizes $\sum_{i=1}^{3}\left|x_i - p\right|$

i. Argue that if p is absolute-optimal, then $p \in [x_1, x_3]$

ii. Use this to derive an absolute-optimal p.

(c) Now suppose that n agents, $\{1,2,3,\ldots,n\}$, live on this street where $x_1 \leq x_2 \leq x_3 \ldots \leq x_n$ and n is an odd number. Derive an absolute-optimal p

Question 8

Two random variables x_1 and x_2 are uniformly drawn from [0,1] Define the following function:

$$G(p) = p \times \text{ Probability } \left[p \geq \max\left(x_1, x_2\right)\right] \quad \forall p \in [0,1]$$

(a) Derive, with a clear explanation, the expression for $G(p)$.

(b) Plot $G(p)$

(c) Is G convex or concave in p? Give clear explanations for your answer.

(d) Find $\max_{p\in[0,1]} G(p)$

Question 9

Let $X \subset \mathbb{R}$ and $f : X \rightarrow X$ be a continuous function.

(a) Suppose $X = [0,1]$. By using the Intermediate Value Theorem, show that there exists $x^* \in X$ such that $f\left(x^*\right) = x^*$.

(b) In each of the cases below, determine whether there exists $x^* \in X$ such that $f\left(x^*\right) = x^*$. Justify your claim by either providing a proof or a counter-example.

i. $X = (0,1)$ and f is continuous.

ii. $X = [0,1] \cup [2,3]$ and f is continuous.

iii. $X = [0,1]$ but f is not continuous.

(c) Let $f_i : [0,1] \rightarrow [0,1], i = 1,2,\ldots,m$, be a collection of m continuous functions. Prove that there exists $x^* \in [0,1]$ such that $\sum_{i=1}^{m} f_i\left(x^*\right) = mx^*$

ISI PEB 2020

Question 1

Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with a uniform distribution on $(0, \theta]$ where $\theta > 0$.

(a) Write down the joint probability density function of $X_1, \ldots X_n$

(b) Suppose x_i is a realization of X_i, for each $i = 1, \ldots, n$ And suppose the value of θ is unknown. Find the value of θ that maximizes the joint p.d.f. in part (a) given that $x_1, \ldots, x_n$ have been observed. (This is called the maximum likelihood estimate of θ.)

(c) Consider the function: $f(x, y) = x^2 + y^2 - 2x$

i. Find the maximum value of f over the region $\left\{(x, y) \mid 2x^2 + 3y^2 - 2x \leq 100\right\}$

ii. Find the minimum value of f over the region $\left\{(x, y) \mid 2x^2 + 3y^2 - 2x \geq 100\right\}$

Question 2

A tournament consists of n players and all possible $C(n, 2) = \frac{n(n-1)}{2}$ pairwise matches between them. There are no ties in a match: in any match, one of the two players wins. The score of a player is the number of matches she wins out of all her $(n-1)$ matches in the tournament. Denote the score vector of the tournament as $s \equiv (s_1, \ldots, s_n)$ and assume without loss of generality $s_1 \geq s_2 \geq \ldots \geq s_n$

(a) For any $2 \leq k \leq n$, show that $s_1 + \ldots + s_k \geq C(k, 2)$, where $C(k, 2) = \frac{k(k-1)}{2}$

(b) Suppose $n > 3$ and players 1,2,3 win every match against players in $\{4, \ldots, n\}$. Find the value of $s_4 + \ldots + s_n$?

(c) Suppose $s_n = s_0, s_{n-1} = s_0 + 1, s_{n-2} = s_0 + 2$ for some positive integer s_0 and $n \geq 3$. Show that

$$s_0 \leq \frac{(n-2)(n-3)}{2n}$$

(d) A tournament generates a score vector s such that

$$s_j - s_{j+1} = 1 \text{ for all } j \in \{1, \ldots, n-1\}$$

What is the score vector of this tournament? For every Player j, who does Player j beat in this tournament?

(e) Suppose there are six players, i.e., $n = 6$. There is a tournament such that each player has a score of at least two and difference in scores of any two players is not more than one. What is the score vector of this tournament? Construct a tournament (describing who beats who) which generates this score vector.

Question 3

Consider the following equation in x

$$(x-1)(x-2)\cdots(x-n) = k \tag{1}$$

where $n > 1$ is a positive integer and k is a real number. Argue whether the following statements are true or false by providing a proof or a counter example.

(a) Suppose $n = 2$. There is a real solution to Equation (1) for every value of k

(b) Suppose $n = 3$. There is a real solution to Equation (1) for every value of k.

(c) For all $k \geq 0$ and for every positive integer $n > 1$, there is a real solution to Equation (1).

(d) For all $k < 0$ and for every odd positive integer $n > 1$, there is a real solution to Equation (1)

(e) For all $k < 0$, there is some even positive integer n such that a real solution to Equation (1) exists.

Question 4

Consider an economy inhabited by identical agents of size 1. A representative agent's preference over consumption (c) and labour supply (l) is given by the utility function

$$u(c, l) = c^{\alpha}(24 - l)^{1-\alpha}, 0 < \alpha < 1$$

Production of the consumption good c is given by the production function $c = Al$, where $A > 0$ is the productivity of labour. Both the commodity market and labour market are perfectly competitive: the buyers and sellers take the price as given while taking demand and supply decisions. Let us denote the hourly wage rate by $w > 0$ and price of the consumption good by $p > 0$

(a) Competitive Equilibrium: A competitive equilibrium is given by the allocation of consumption and labour, (c^{CE}, l^{CE}) , and the relative price ratio, $\frac{w}{p}$, such that, given w and p, a representative agent decides her labour supply, l^S, and consumption demand, c^D, to maximize her utility; a firm decides its labour demand, l^D, and supply of consumption good, c^S, to maximize its profit; and, finally, both the commodity market and labour market clear, that is, $l^D = l^S$ and $c^D = c^S$

i. Set up the representative agent's utility maximization problem. Write down the first order conditions for this maximization problem and determine l^S and c^D as functions of w and p

ii. Set up a firm's profit maximization problem. Determine l^D and c^S as functions of w and p

iii. Determine the competitive equilibrium allocation, (c^{CE}, l^{CE}) and the relative price ratio, $\frac{w}{p}$.

(b) Pareto efficient allocation: For this economy define the concept of a Pareto efficient allocation of consumption and labour. Find out a Pareto efficient allocation of consumption and labour in this economy. Provide a clear explanation.

Question 5

(a) Ms. A's income consists of Rs. 1,00,000 per year from pension plus the earnings from whatever she sells of the 2.000 kilograms of rice she harvests annually from her farm. She spends this income on rice (x) and on all other expenses (y). All other expenses are measured in Rupees, so that the price of y is Rs. 1. Last year rice was sold for Rs. 20 per kilogram, and Ms. A's rice consumption was 2,000 kilograms, just the amount produced on her farm. This year the price of rice is Rs. 30 per kilogram. Ms. A has standard convex preferences over rice and all other expenses. Answer the following two questions without referring to any utility function or indifference curves.

i. What will happen to her rice consumption this year increase, decrease, or remain the same? Give a clear explanation for your answer.

ii. Will she be better or worse off this year compared to last year? Explain clearly.

(b) There are two goods a and y. Mr. B has standard convex preferences over the two goods. He has endowments of $e_x > 0$ units of good x and $e_y > 0$ units of good y. He does not have any other source of income. When the price of good y is Rs. 1 and the price of good x is Rs. p_x he decides neither to buy nor to sell good x.

i. Suppose that, for good x, the prices have become Rs. $p_L < p_x$ if an individual is a seller and Rs. $p_H > p_x$ if an individual is a buyer. The price of good y remains Rs. 1 no matter whether an individual buys or sells good y. Write down the equation of the new budget constraint and draw it labelling the important points clearly.

ii. Will Mr. B buy or sell good x ? By how much? Give a clear explanation for your answer without referring to any utility function or indifference curves.

Question 6

Consider a moneylender who faces two types of potential borrowers: the safe type and the risky type. Each type of borrower needs a loan of the same size L to invest in some project. The borrower can repay only if the investment provides sufficient returns to cover the repayment. Suppose the safe type is always able to secure return R from the investment, where $R > L$. On the other hand, the risky type is an uncertain prospect, he can obtain a higher return R' (where $R' > R$) but only with probability p. With probability $1 - p$, the investment backfires and he gets a return of zero. The money lender has enough funds to lend to just one applicant and there are two of them, one risky and one safe. Each borrower knows his own type, but the moneylender does not know the borrower's type. He just knows that one is safe and the other is risky. Since the moneylender has enough funds to lend to just one applicant, when both the borrowers apply for the loan, he gives the loan randomly to one of them say by tossing a coin. Assume that the lender supplies the loan from his own resources and his opportunity cost is zero.

(a) What is the highest interest rate, call it i_s, for which the safe borrower wants the loan? What is the highest interest rate, i_r, for which the risky borrower wants the loan? Who is willing to pay a higher interest rate, the risky borrower or the safe borrower?

(b) The lender's objective is to maximize his expected profit. Argue clearly that the lender's effective choice is between two interest rates, i_s and i_r (That is, argue that the lender will not choose any interest rate strictiy lower than min $\{i_r, i_s\}$, any interest rate strictly higher than max $\{i_r, i_n\}$, or any interest rate strictly in between i_r and i_n.)

(c) Argue that when the lender charges i_r, his expected profit is given by $p\left(1+i_r\right)L-L$. Derive, with a clear argument, the expression of lender's expected profit when he charges i_s

(d) An equilibrium with credit rationing occurs when, at the equilibrium interest rate, some borrowers who want to obtain loans are unable to do so; however, lenders also not raise the interest rate to eliminate the excess demand. Explain clearly that we have an equilibrium with credit rationing when

$$p<\frac{R}{2R'-R}$$

Question 7

Consider an economy where identical agents (of mass 1) live for two periods: youth (period 1) and old age (period 2). The utility function of a representative agent born at time t is given by

$$u\left(c_{1,t}, c_{2,t+1}\right)=\log\left(c_{1,t}\right)+\beta\log\left(c_{2,t+1}\right)$$

where c_1 denotes consumption in youth, c_2 denotes consumption in old age, and $0<\beta<1$ is the discount factor reflecting her time preference. In her youth the representative agent supplies her endowment of 1 unit of labour inelastically and receives the market-determined wage rate w_1. So in her youth the agent faces the budget constraint $c_{1,t}+s_t=w_0$ where s_t denotes her savings. When old, she just consumes her savings from youth plus the interest earning on her savings, $s_t r_{t+1}$, where r_{t+1} is the market-determined interest rate in period $t+1$. That is, when old, her budget constraint is $c_{2,t+1}=\left(1+r_{t+1}\right)s_t$

(a) Set up the agent's utility maximization problem by showing her choice variables clearly.

(b) Write down the first order conditions for this maximization problem and derive the savings function. Explain how savings. s_t. if it does, depends on the interest rate r_{t+1}

The production function of the economy is given by $Y_t=AK_t^\alpha L_t^{1-\alpha}$ $0<\alpha<1$, where K and L denote the amounts of capital and labour in the economy, respectively. Capital depreciates fully after use, that is. the rate of depreciation of capital is one. Factor markets being competitive, the equilibrium factor prices are given by their respective marginal products.

(c) Derive the equilibrium wage rate $\left(w_t\right)$ of the economy in terms of K_t.[Keep in mind that the mass of agents is 1 and each agent supplies her endowment of 1 unit of labour inelastically.]

The role of the financial sector (banks, stock market, and so on) is to mobilize the savings of households to bring it for effective use by the production sector. But the financial sector does not work well and a fraction $0<\theta<1$ of aggregate savings gets lost (vanishes in thin air) in the process of intermediation.

(d) Derive the law of motion of capital (that is. express capital in period $t+1$, K_{t+1}, in terms of capital in period t, K_t)

(e) Derive the steady state anount of capital of the economy.

(f) How does the steady state amount of capital depend on the inefficiency of the financial sector θ?

Question 8

Consider a Solow-Swan model with learning by doing. Assume that the production function is of the form

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}$$

where A is the level of technological progress and grows at the rate $g > 0$, L is the population with grows at the rate $n > 0$, K is the capital stock, Y is GDP, and $\alpha \in [0, 1]$. Assume that

$$\dot{K} = sY - \delta K$$

Define $Z = \frac{K}{AL}$ as the capital labor ratio in efficiency units. Let output per worker be given by $Q = AZ^{\alpha}$. The parameter $s \in [0, 1]$ denotes the savings rate. The parameter $\delta \in [0, 1]$ denotes the depreciation on capital.

(a) Derive an expression for $\frac{\dot{Z}}{Z}$

(b) Instead of assuming that the rate of technological progress is constant (g), now assume that the instantaneous increase in A is proportional to output per worker, i.e., there is learning by doing

$$\dot{A} = \gamma Q$$

Show that the law of motion of capital is given by

$$\dot{Z} = (s - \gamma Z)Z^{\alpha} - (\delta + n)Z$$

(c) Draw a diagram describing the dynamics of growth in the model with learning by doing. Plot Z on the $x-$ axis, and the appropriate functions on the $y-$ axis

(d) In contrast to the model with no learning by doing, does an increase in the investment rate raise the balanced-growth rate? What does this tell you about the change in policy having level effects versus growth effects in the with learning by doing in contrast to the model when there is no learning by doing? Show your answer using the diagram in part (c).

Question 9

Suppose households who live till T periods maximize $\sum_{n=t}^{T} \beta^{n-t} \ln(c_n)$ Where c_n represents their income in period $n = t, t+1, 1+2 \ldots .T$ and β is a parameter with $0 < \beta < 1$. Suppose per period income and the saving of households are y_n and s_n respectively and the activity starts from the beginning of their life t. Further, the net interest rate on saving in between any two periods is exogenously fixed at r and so the gross rate of return is $1 + r$. Households have only two activities in every period - consuming and saving.

(a) Write down the sequence of budget constraints (one for each period) and the aggregate budget constraint derived from these periodic budget constraints where on the left hand side, consumption levels for all the periods appear, and on the right hand side, income in all periods appears.

(b) Under what condition between β and r, is the optimal solution for the above problem yield constant consumption, $\widetilde{C}$, in every period?

(c) Suppose the condition that you derive in (b) holds. Then answer the following questions

i. For a transitory change in income in period t only, calculate the change in the constant level of consumption, $\tilde{C}$.

ii. For a transitory change in income in period $t + k$ only; calculate the change in the constant level of consumption, $\widetilde{C}$.

(d) For a permanent change in income (assume the same amount of income change in all periods), calculate the change in the constant level of consumption, $\tilde{C}$. Compare this value derived with part c (i).

ISI PEB 2021

Question 1

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function which has at least three distinct zeros. (We say x is a zero of f if $f(x) = 0$). Let $g : \mathbb{R} \to \mathbb{R}$ be defined as follows: $g(x) = e^{\frac{x}{2}} f(x)$ for all $x \in \mathbb{R}$.

(a) Prove that g has at least three distinct zeros.

(b) Prove that the function $f + 2f'$ has at least two distinct zeros.

Question 2

(a) Consider the following two variable optimization problem:

$$\max_{x,y} \left(x^2 + y^2\right)$$

subject to

$$x + y \leq 1$$
$$x, y \geq 0$$

Find all solutions of this optimization problem.

(b) In a kingdom far, far away, a King is in the habit of inviting 1000 senators to his annual party. As a tradition, each senator brings the King a bottle of wine. One year, the Queen discovers that one of the senators is trying to assassinate the King by giving him a bottle of poisoned wine. Unfortunately, they do not know which senator, nor which bottle of wine is poisoned, and the poison is completely indiscernible. However, the King has 10 prisoners he plans to execute. He decides to use them as taste testers to determine which bottle of wine contains the poison. The poison when taken has no effect on the prisoner until exactly 24 hours later when the infected prisoner suddenly dies. The King needs to determine which bottle of wine is poisoned by tomorrow so that the festivities can continue as planned. Hence he only has time for one round of testing. How can the King administer the wine to the prisoners to ensure that 24 hours from now he is guaranteed to have found the poisoned wine bottle?

Question 3

(a) Let A and B be matrices for which the product AB is defined. Show that if the columns of B are linearly dependent, then the columns of AB are linearly dependent.

(b) Let e_i denote the column vector with three elements, each of which is zero, except for the i-th element, which is 1. Consider a linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$ with $L(e_1) = e_1, L(e_2) = e_1 + e_2, L(e_3) = e_2 + e_3$. Does L map $\mathbb{R}^3$ onto $\mathbb{R}^3$?

Question 4

This question pertains to a situation in which a particular commodity, like rice, is both available at a subsidised rate from a fair price shop (ration shop) and at a higher price from the open market. Suppose a consumer can buy a certain (fixed) quantity of rice at a lower price from the ration shop (that is, there is a ration quota). In addition, he can buy more of rice (assume a uniform quality of rice) from the open market at a higher price. (You may assume that consumers preferences are represented by standard downward sloping, smooth, convex indifference curves.)

(a) Graphically depict the consumer's equilibrium (assuming he exhausts the ration quota and in addition buys from the open market).

(b) Suppose rice is a normal good. What will happen to the quantity of rice purchased from the open market (over and above the ration quota) in equilibrium if there is a cut in the ration quota? Briefly explain.

(c) Suppose rice is a normal good. What will happen to the quantity purchased in the open market (over and above the ration quota) if the subsidised price (price at which the ration quota rice could be bought) is increased (but is still lower than the open market price)? Will your conclusion change if rice is an inferior good? Briefly explain.

Question 5

There are plenty of fish in the Dull Lake. Boats can be hired by fishermen to catch fish and sell it on the fish market. The revenue earned each month from a total of x boats is given by the following expression: Rupees $10,000\{4x - \frac{x^2}{2}\}$. Each boat costs Rupees $20,000$ each month.

(a) Derive the marginal and average revenue per boat.

(b) The Dull municipality is considering giving out permits for each boat so that they can track who is fishing from the lake. If these permits are allocated freely, how many boats will fish every month?

(c) If total profit is to be the maximum possible, how many boats should fish every month?

(d) Dull municipality decides to charge for the permits instead of giving them out for free. What should the per-boat charge for the permit be if total profits are to be the maximum possible?

Question 6

The Shoddy Theater screen movies every week and is located on a university campus which has only students and faculty as residents. It is the only source of watching movies for both faculty and students, and is large enough to accommodate all faculty and students. Faculty demand for movie tickets is given by $500 - 4P_F = Q_F$, where P_F refers to the price of ticket paid by faculty and Q_F refers to the number of tickets purchased by faculty. Demand by students is described by $100 - 2P_S = Q_S$, where P_S refers to the price of ticket paid by students and Q_S refers to the number of tickets purchased by students. The cost to service demand equals 500.

(a) If the price charged is to be the same for faculty and students, what price would Shoddy Theater set in order to maximize its profits?

(b) Now imagine that Shoddy Theater decides to charge different prices for faculty and students. What would these prices be, if Shoddy Theater wants to maximize profits?

Question 7

Consider a Solow type economy, producing a single good, according to the production function:

$$Y(t) = K(t)^{\alpha} L(t)^{(1-\alpha)}$$

Where, $0 < \alpha < 1$, and $Y(t)$, $K(t)$ and $L(t)$ are the output of the good, input of capital, and input of labour used in the production of the good, respectively, at time t. Capital and labour are all fully employed. Labour force grows at the exogenous rate, $\eta > 0$, i.e.

$$\frac{1}{L(t)}\frac{dL(t)}{dt} = \eta > 0$$

Part of the output is consumed and part saved. Let $0 < s < 1$ be the fraction of output that is saved and invested to build up the capital stock. Also assume that there is no depreciation of capital stock. Then it follows that:

$$sY(t) = \frac{dK(t)}{dt}$$

Where $\frac{d}{dt}$ is the time derivative. With this above given description of the economy, one can find out the steady state growth rate of Y, for this economy. Growth rate of output is given by: $\frac{1}{Y(t)}\frac{dY(t)}{dt} \equiv g_Y$. Assume, the economy begins at date 0, from a per capita capital stock, $k(0) \equiv \frac{K(0)}{L(0)} < k^*$, where k^* denotes the steady state per capita capital stock.

(a) Demonstrate formally, whether the growth rate of output (g_Y), at the beginning date 0, is greater, equal or less than the steady state growth rate of output.

(b) For the same economy, consider, two alternative beginning date scenarios: with per capita capital stock, given by: Case 1: $k(0)$; Case 2: $k'(0)$ where $k(0) < k'(0) < k^*$. Can you compare the beginning date growth rates of output in the new cases?

(c) Next, consider two Solow type economies, namely, A and B. They are isolated from each other and are working on their own. Both the economies have absolutely the same description as given before, except for the fact that the fraction of income saved in Country A, denoted by s_A is greater than the fraction of income saved in Country B, denoted by s_B. Let $k^A(0)$ be the initial date per capita capital stock in country A and $k^B(0)$, the initial date per capita capital stock in country B, which are both less than their respective steady state values. Assume $K^A(0) < K^B(0)$. Can you figure out whether the initial date growth rate of out-put in country A is greater than, equal or less than the initial date growth rate of output in country B? In case you find the data provided to you is insufficient to make any comment on this, please point it out.

Question 8

(a) What is the money multiplier? What determines its size? What is the relationship between the monetary base, the money multiplier, and the money supply. Which of these variables can the central bank change to change the money supply. What is the direction of change in each case?

(b) Why might the cash/deposit ratio and the reserve to asset ratio be decreasing functions of the rate of interest? How dues an interest-sensitive money supply affect the LM-curve? Illustrate with a diagram, comparing this LM curve with the standard LM. How does this change the effectiveness of counter-cyclical fiscal policy (in a closed eocnomy)? Explain.

Question 9

(a) What is the difference between the real and the nominal exchange rate? Give an example to explain this to someone who has not studied economics. Is an increase in the real cost of imports an improvement or a deterioration in the terms of trade?

(b) A small open economy has a government budget surplus and a trade deficit. Explain whether there is a private sector surplus, deficit or balance. Examine the consequences in the short run for output, the trade balance and the government budget balance of a sudden fall in private consumption in this economy (due to an epidemic in the small country) under (a) fixed exchange rates, (b) flexible exchange rates. Use the Mundell-Fleming model with perfect capital mobility. Explain the adjustment mechanisms.

ISI PEB 2022

Question 1

Answer the following questions.

(a) Find all maxima and minima of the function $f(x, y) = xy$, subject to the constraints $x + 4y = 120$ and $x, y \geq 0$.

(b) Find the points on the circle $x^2 + y^2 = 50$ which are closest to and farthest from the point $(1, 1)$.

(c) For what values of α are the vectors $(0, 1, \alpha), (\alpha, 1, 0)$ and $(1, \alpha, 1)$ in $\mathcal{R}^3$ linearly independent?

Question 2

Let $f : \mathcal{R} \to \mathcal{R}$ be a continuous function.

(a) Let Q denote the set of rational numbers. Prove that, if $f(Q) \subseteq \{1, 2, 3, \ldots\}$, then f is a constant function.

(b) Calculate the value of $f'(0)$ when f is differentiable and $|f(x)| \leq x^2$ for all $x \in \mathcal{R}$.

Question 3

Suppose a set of $N = \{1, 2, \ldots, n\}$ political parties participated in an election; $n \geq 2$. Suppose further that there were a total of V voters, each of whom voted for exactly one party. Each party $i \in N$ received a total of V_i votes, so that $V = \sum_{i=1}^{n} V_i$. Given the vector $(V_1, V_2, \ldots, V_n)$, whose elements are the total number of votes received by the n different parties, define $P_1 (V_1, V_2, \ldots, V_n)$ as the probability that two voters drawn at random with replacement voted for different parties and define $P_2 (V_1, V_2, \ldots, V_n)$ as the probability that two voters drawn at random without replacement voted for different parties. Answer the following questions.

(a) Derive the ratio $\frac{P_2}{P_1}$ as a function of V alone.

(b) Consider the special case where $V_i = \frac{V}{n}$ for all $i \in N$. For this case, find the probabilities P_1 and P_2.

Question 4

Consider an agent living for two periods, 1 and 2. The agent maximizes lifetime utility, given by:

$$U(C_1) + \frac{1}{(1+\rho)} U(C_2),$$

where $\rho > 0$ captures the time preference, while C_1 and C_2 are the agent's consumption in period 1 and period 2, respectively. The agent supplies one unit of labor inelastically in period 1, earning a wage w. A portion of this wage is consumed in period 1 and rest is saved (denoted s). In period 2 the agent does not work, but receives interest income on the savings. Principal plus the interest income on savings goes to finance period 2 consumption. Thus, $C_1 + s = w$ and $C_2 = (1 + r)s$, where r is the rate of interest.

Assume that the per period utility function can be represented by (and only by) any positive linear transformation of the form $U(C) = \frac{C^{1-\theta}-1}{1-\theta}$, where $0 < \theta < 1$.

(a) Demonstrate, deriving your claim, how optimal savings, s, would respond to changes in r.

(b) Now suppose, initially, $r = \rho$. What happens to optimal savings, s, if r and ρ increase by the same amount (so that the condition $r = \rho$ continues to hold)?

Question 5

A profit maximizing monopolist produces a good with the cost function $C(x) = cx, c > 0$, where x is the level of output, $x \geq 0$. It sells its entire output to a single consumer with the following utility function:

$$u(y) = \theta\sqrt{y} - T(y)$$

where y is the amount of the good purchased by the consumer and T is the payment made by the consumer to the monopolist to purchase the output; $0 \leq y \leq x; \theta > 0$. Suppose

$$T(y) = py + t;$$

where $p \geq 0, t \geq 0$ if $y > 0$, and $T(0) = 0$. Thus, in order to purchase any positive amount of the good, the consumer may have to pay a lump-sum amount t, or a per unit price p, or both.

(a) Find the profit of the monopolist when it can choose any non-negative combination of t and p.

(b) Find the profit of the monopolist when it can choose any non-negative p, but is forced to set $t = 0$. Calculate how this profit relates to the profit derived in part (a) and explain your result.

(c) Calculate when social surplus is higher, explaining your result.

(d) Calculate when consumer's surplus is higher, explaining your result.

Question 6

Answer the following questions.

(a) Let the input demand functions of a profit-maximizing competitive firm operating at unit level of output be given by:

$$x_1 = 1 + 3w_1^{-(1/2)}w_2^a \text{ and } x_2 = 1 + bw_1^{(1/2)}w_2^c;$$

where w_1 and w_2 are input prices. Find the values of the parameters a, b and c.

(b) Check whether the following data, summarizing the observed input-output choices of a competitive firm under three different output-input price situations, are consistent with the hypothesis of profit maximization by that firm.

	p	w	q	x
Observation 1	50	20	20	25
Observation 2	45	15	24	36
Observation 3	40	20	16	16

Here p and w denote output price and input price, respectively, while q and x denote, respectively, the units of output supplied and input demanded by the firm. Each of the three rows specifies an observation of the output-input price configuration and the output-input choice of the firm under that particular price configuration.

(c) Suppose, for the production function $f(x_1, x_2)$, the cost function of a competitive firm is $c(q; w) = w_1^a w_2^{1-a} q$, where $w = (w_1, w_2)$ is the input price vector and q is the level of output; $\alpha \in (0, 1)$. Derive the conditional input demand functions and the production function of the firm.

Question 7

Consider a world economy consisting of Home(H) and Foreign (F). Each of these countries produces a single good that is both consumed domestically and exported. Let Foreign output be the numeraire and let p be the relative price of the H produced good. Assume full employment in both countries, so that H produces a fixed output Y and F produces a fixed output Y^*. Let E be the Home expenditure in terms of its own good and let E^* be the Foreign expenditure measured in terms of the foreign good. We will treat E as a parameter of the model, while E^* is endogenous. Assume that consumers have Cobb-Douglas utility functions with fixed expenditure shares. Let α be the share of expenditure of Home consumers on the Foreign produced good and let α^* be the share of expenditure of Foreign consumers on the Home produced good. Assume further $1 - \alpha > \alpha^*$ (i.e., the expenditure share of Home consumers on the Home produced good is greater than the expenditure share of Foreign consumers on the Home produced good). World income equals world expenditure, and goods markets clear.

Now, suppose E falls.

(a) What will happen to p ?

(b) What will happen to the trade balance of Home, denominated in units of the Foreign good (i.e., to $p(Y - E)$)?

Prove your claims.

Question 8

Consider the Solow growth model with constant average propensity to save s, labor supply growth rate n, no technological progress and zero rate of depreciation. Let v denote the capital-output ratio.

(a) Prove that, at the steady state, $\frac{s}{v} = n$.

(b) Now suppose that, in some initial situation, $\frac{s}{v} > n$. Explain how market forces will operate to restore, over time, the equality $\frac{s}{v} = n$.

(c) In the process of adjustment in (b), in which direction will the real wage and real rental on capital change? Explain.

Question 9

Answer the following questions.

(a) Using a simple Keynesian model of income determination, derive and explain the conditions under which a rise in the marginal propensity to save will reduce aggregate savings in the economy.

(b) Using a model of aggregate demand and aggregate supply, exiplain how an increase in fuel prices would impact aggregate output, employment and the price level.

ISI PEB 2023

Question 1

Consider a used car market with 600 buyers each willing to buy exactly one used car, and 500 sellers each having exactly one used car. Out of the 500 used cars, 400 are of good quality (peaches) and 100 are of bad quality (lemons). The monetary valuation of owning a peach is Rs. 100 for a buyer and Rs. 50 for a seller. On the other hand, the monetary valuation of owning a lemon is Rs. 10 for both a buyer and a seller. A seller knows whether the car she owns is a peach or a lemon, whereas a buyer only knows that there are 400 peaches and 100 lemons. Both the buyers and the sellers know the various valuations.

(a) What outcome maximizes the aggregate surplus of the economy? Provide a clear explanation for your answer.

(b) i. Derive, with a clear explanation, the supply of used cars as a function of price. Draw this supply curve by plotting number of used cars on x-axis and price on y-axis. [You must label all the important points in the figure clearly.]

ii. Derive, with a clear explanation, the demand for used cars as a function of price. Draw this demand curve in the same figure as in part (i). [You must label all the important points in the figure clearly.]

iii. Use the demand and supply functions above to find out all possible competitive equilibria in the used car market mentioning clearly which types of car, lemon or peach, are bought and sold in each equilibrium.

(c) Now suppose that buyers also know the identity of all cars, that is, whether any given car is a peach or a lemon. Use a similar demand-supply analysis as above to solve for all possible competitive equilibria in the used car market in this scenario.

Question 2

Consider an industry with 2 firms - a private firm (indexed by r) and a public firm (indexed by u) - producing a homogeneous product and competing in quantities. The firms face an inverse demand function $p = a - bQ, a > 0, b > 0$, where $Q = q_r + q_u$ denotes aggregate output, and q_r and q_u denote the amounts of output produced by the private and public firms respectively. Each firm i faces the total cost of production $cq_i, i = r, u, 0 < c < a$.

(a) For any q_r and q_u, derive the expressions for (i) private firm's profit, (ii) public firm's profit, (iii) consumer surplus, and (iv) welfare (sum of consumer surplus and producer surplus).

(b) The private firm's objective is to maximize its own profit. For a given q_u, set up the private firm's maximization problem and derive its optimal choice of output q_r. [This exercise gives you the reaction function of the private firm.]

(c) The public firm's objective is to maximize welfare. For a given q_r, set up the public firm's maximization problem and derive its optimal choice of output q_u. [This exercise gives you the reaction function of the public firm.]

(d) Recall that the two firms compete in quantities.

i. Define the concept of equilibrium in this context and find out the amounts of output, q_r^* and q_u^*, the two firms produce in equilibrium. Find out the expressions of price, profits of the two firms, consumer surplus and welfare in the equilibrium.

ii. Illustrate this equilibrium by drawing the two reaction functions you have derived in parts (b) and (c) (plot q_u in x-axis and q_r in y-axis). [You must label the important points in the figure clearly.]

(e) Suppose that the marginal cost of the private firm falls to $c_r < c$ while the marginal cost of the public firm remains the same at c. Draw the new reaction functions and explain clearly how the following outcomes change in the new equilibrium (as compared to the old equilibrium): q_r, q_u, Q, price, profits of the two firms, consumer surplus and welfare. [There is no need to derive the exact expressions; just qualitative answers are enough.]

Question 3

There is a unit mass of consumers all of whom want to purchase at most 1 unit of a good. Consumer v has a valuation v for this good, where $v \in [0, 1]$. Assume that v is uniformly distributed over interval $[0, 1]$ so that the number of consumers with valuation in between a and b, where $0 \leq a < b \leq 1$, is $b - a$. There is a monopoly firm with total cost of producing q units of the good given by $\frac{q}{3}$. The firm does not know the identity of any consumer and hence must charge a uniform price to all the consumers.

(a) For any price p, derive, with a clear explanation, the demand facing the monopoly firm.

(b) Derive, with a clear explanation, the monopoly price and profit level.

(c) Suppose that the firm can, for a cost, get to know whether a consumer belongs to the interval $\left[0, \frac{4}{5}\right]$, or to the interval $\left(\frac{4}{5}, 1\right]$. What is the maximum amount the firm is willing to pay for this information? Give a clear explanation for your answer.

Question 4

Consider an aggregate demand and aggregate supply model where, in the short run, aggregate capital is fixed at the level $\bar{K}$. The aggregate demand curve, aggregate output (Y) demanded as a function of aggregate price level (P), is given by a standard downward-sloping curve. The aggregate supply curve, aggregate output (Y) supplied as a function of aggregate price level (P), is not standard, and the question leads you to derive the aggregate supply curve. The aggregate production function is linear in capital and labour $(L) : Y = AL + \bar{K}, A > 0$. The labour union is very powerful and dictates the minimum aggregate nominal wage rate as $\bar{W}$. Each worker is endowed with one unit of labour which they supply inelastically if the producers offer the nominal wage $W > \bar{W}$. A worker does not supply any labour if $W < \bar{W}$. At $W = \bar{W}$, a worker is indifferent between supplying and not supplying her labour endowment. The number of workers available in the economy is fixed at $\bar{L}$.

(a) Derive, with a clear explanation, the aggregate labour supply $\left(L^S\right)$ in this economy as a function of the aggregate nominal wage rate, W.

(b) Note that the marginal product of labour is constant, $A > 0$.

i. Derive, with a clear explanation, the aggregate labour demand $\left(L^D\right)$ in this economy as a function of the real wage rate, $\frac{W}{P}$.

ii. Using your answer to part (i) above, derive the aggregate labour demand $\left(L^D\right)$ in this economy as a function of the aggregate nominal wage rate, W.

(c) Choose an arbitrary aggregate price level, P, and draw the aggregate labour supply $\left(L^S\right)$ and aggregate labour demand $\left(L^D\right)$ curves, as functions of W, by plotting labour (L) on x-axis and nominal wage (W) on y-axis. Think about the labour market equilibrium for the arbitrary aggregate price level P that you have chosen. Note that the equilibrium employment (L^*) in the economy depends on the arbitrary price level P that you choose. Derive, with a clear explanation, the equilibrium employment (L^*) as a function of aggregate price level P.

(d) Derive, with a clear explanation, aggregate output (Y) supplied as a function of aggregate price level (P). Draw this aggregate supply curve by plotting Y on x-axis and P on y-axis.

(e) Recall that the aggregate demand curve is given by a standard downward-sloping curve. Explain the effectiveness of the standard monetary and fiscal policies in this set up.

Question 5

Consider the following version of the Solow growth model. The aggregate output at time t, Y_t, depends on the aggregate capital stock (K_t) and aggregate labour force (L_t) in the following way:

$$Y_t = (K_t)^{\alpha} (L_t)^{1-\alpha}, 0 < \alpha < 1$$

There is perfect competition in the factor market so that, in equilibrium, each factor is paid its marginal product and the total output is distributed to all the households in the form of wage earnings and interest earnings. Households save a proportion $0 < s < 1$ of their disposable income in every period. All household savings are invested which augment the capital stock over time. There is no depreciation of capital. Population and therefore the aggregate labour force grows at a constant rate $n > 0$.

(a) The govermment taxes the interest earnings at the rate $0 < \tau < 1$. Wage earnings are not taxed. The government uses the collected taxes to fund government consumption; in particular, the tax collection is not used for investment at all.

i. Derive, with clear explanations, the expressions for aggregate wage earning, aggregate interest earning and aggregate savings (S_t) of the economy in terms of Y_t.

ii. Define $k_t \equiv \frac{K_1}{L_t}$, the capital-labour ratio in period t. Derive, with a clear explanation, the law of motion of capital-labour ratio, that is, the equation with k_{t+1} on the left-hand side and k_t on the right-hand side.

iii. Derive, with a clear explanation, the steady-state level of capital-labour ratio in this economy, k^*, and examine how k^* changes with changes in the tax rate τ.

(b) As in part (a) above, the government continues taxing interest earnings at the rate τ and wage earnings are not taxed. But consider now that the tax revenue collected is used to fund investment by the government so that the capital stock is furthe: angmented by this public investment.

i. Derive, with a clear explanation, the expression for aggregate investment in this economy.

ii. Derive, with a clear explanation, the new law of motion of capital-labour ratio.

iii. Derive the new steady-state level of capital-labour ratio in this cconomy, k^{**}, and compare it with k^{*}. Does the comparison make economic sense?

iv. How does k^{**} change with changes in τ ? Compare with the response of k^{*} and explain the economic reason behind the differential impact.

Question 6

Consider an individual who lives for two periods. In the first period, she earns an wage income W and takes her consumption savings decision once income is realized. In the second period, she has no wage income but receives the return along with her principal amount of savings, s. The gross rate of interest is $R > 1$. Suppose that there is a government which collects an amount T in the form of lumpsum tax from the wage income in the first period (this can be considered as mandatory savings of the individual) and returns the amount T in the form of a lumpsum transfer in the second period. Suppose that the utility derived by the individual who consumes c_1 in the first period and c_2 in the second period is given by $u(c_1) + \beta u(c_2)$, where β is a discount factor with $0 < \beta < 1$ and the utility function u is strictly increasing and strictly concave.

(a) Set up the individual's utility maximization problem by specifying her budget constraint clearly. Derive the first-order condition of this utility maximization problem by showing your procedure clearly. Provide a clear economic interpretation of the first-order condition.

(b) Note that $\beta < 1$ implies that the individual is myopic (shortsighted), she puts less weight on future period. Explain intuitively whether a more myopic individual will save more or less than a less myopic individual. Verify your intuition by determining the sign of $\frac{ds}{d\beta}$.

(c) Note also that the individual's personal savings, s, depends on the government mandated savings T. Explain intuitively whether the government mandated savings increases or decreases personal savings. Verify your intuition by determining the sign of $\frac{ds}{dT}$

(d) One rupee received in benefits in period 2 would require an individual to save an amount $\frac{1}{R}(< 1 \text{ since } R > 1)$ in period 1. Explain intuitively whether the government mandated savings make the individual cut back her personal savings at a rate higher or lower than $\frac{1}{R}$. Verify your intuition.

(e) In the light of your answer to part (b) and from the expression of $\frac{ds}{dT}$ you expect that the rate of change in personal savings in response to a change in T depends on the discount factor β. Prove that more myopic individuals reduce their personal savings at a higher rate.

Question 7

Consider the following feasible region C in $\mathbb{R}^2$ for an optimization program:

$$C := \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1, y \leq x\}$$

(a) Suppose $a \neq 0, b \neq 0$. Show that an optimal solution to

$$\max_{(x,y)\in C} ax + by$$

is either $(0,0), (1,0), (1,1)$. Describe all possible values of a and b for which each of $\{(0,0), (1,0), (1,1)\}$ is an optimal solution.

(b) Suppose a and b are independently drawn from $[-1, 1]$ using a probability distribution with cumulative distribution function (cdf) F. What is the probability that the unique optimal solution to the above optimization problem is $(1, 0)$?

Question 8

Suppose A, B, C, a, b, c are real numbers and $A \neq 0, a \neq 0$. Suppose for all real values of x, the following holds:

$$\left|ax^2 + bx + c\right| \leq \left|Ax^2 + Bx + C\right|$$

Suppose $B^2 - 4AC > 0$.

(a) Argue that $|A| \geq |a|$.

(b) Show that $b^2 - 4ac > 0$.

(c) Show that $B^2 - 4AC \geq b^2 - 4ac$.

Question 9

Let $\mathcal{F}$ be a class of functions from $[0, \infty)$ to $[0, \infty)$ with the following properties:

- The functions $f_1(x) = e^x - 1$ and $f_2(x) = \ln(x + 1)$ are in $\mathcal{F}$.
- If $f(x)$ and $g(x)$ (not necessarily distinct) are in $\mathcal{F}$, then the functions $f(x) + g(x)$ and $f(g(x))$ are in $\mathcal{F}$.
- If $f(x)$ and $g(x)$ are in $\mathcal{F}$ and $f(x) \geq g(x)$ for all $x \in [0, \infty)$, then $f(x) - g(x)$ is in $\mathcal{F}$.

(a) For any positive integer n, show that $h(x) = nx$ is in $\mathcal{F}$.

(b) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the functions $\ln(f(x) + 1)$ and $\ln(g(x) + 1)$ are in $\mathcal{F}$.

(c) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the function $f(x)g(x) + f(x) + g(x)$ is in $\mathcal{F}$.

(d) If $f(x)$ and $g(x)$ are in $\mathcal{F}$, show that the function $f(x)g(x)$ is in $\mathcal{F}$.

ISI PEB 2024

Question 1

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Show that there always exists a point $c > 1$ such that $f(2) - 2f(1) = cf'(c) - f(c)$.

(b) Let $f : (a, b) \to \mathbb{R}$ be a function such that there is some $c \in (a, b)$ satisfying $f(c) = \max\{f(x) : x \in (a, b)\}$, where $(a, b) = \{x \in \mathbb{R} : a < x < b\}$. Assume that one-sided derivatives $f'_-(c)$ and $f'_+(c)$ exist. Show that $f'_-(c) \geq 0$ and $f'_+(c) \leq 0$

(c) Let f be a function which is continuous on $[0, 1]$ and differentiable on $(0, 1)$, with $f(0) = f(1) = 0$. Assume that there is some $c \in (0, 1)$ such that $f(c) = 1$. Prove that there exists some $x_0 \in (0, 1)$ such that $\left|f'(x_0)\right| > 2$.

Question 2

(a) A restaurant has a menu card containing three Chinese items, six Indian items and five Continental items. Six items are selected at random. Let X and Y denote respectively the number of Indian and Continental items selected. Compute the conditional probability mass function of X given that $Y = 3$. Also compute $\mathbb{E}(X \mid Y = 3)$.

(b) Let the joint probability mass function of X and Y be

$$p(1,1) = \frac{1}{9}, \quad p(2,1) = \frac{1}{3}, \quad p(3,1) = \frac{1}{9}$$
$$p(1,2) = \frac{1}{9}, \quad p(2,2) = 0, \quad p(3,2) = \frac{1}{18}$$
$$p(1,3) = 0, \quad p(2,3) = \frac{1}{6}, \quad p(3,3) = \frac{1}{9}$$

where $p(x, y) = P(X = x, Y = y)$ for all $x = 1, 2, 3$ and $y = 1, 2, 3$. Find the correlation between X and Y. Also find $E(X \mid Y = 3)$.

Question 3

A consumer consumes two goods: visits to a nearby park (x) and a composite consumption good (y), according to preferences $u = xy$. The consumer's income is R and the price of the composite good is normalized to 1. Initially, there is an entry fee p^* per park visit. Suppose the authorities are considering a proposal to reduce the per visit entry fee from p^* to p'.

(a) Find the increase in consumer's surplus resulting from this proposal.

(b) Now suppose there is an alternative proposal to maintain the per visit entry fee at the initial level p^*, but hand out a one-off cash voucher to the consumer (i.e., the consumer would get the cash voucher only once, regardless of how many times she visits the park). Find the minimum value of the cash voucher that would make the consumer not worse off under this alternative proposal, compared to the earlier proposal to reduce the entry fee from p^* to p'.

(c) Next, suppose there is a third proposal which simultaneously reduces the per visit entry fee from p^* to p', and charges a one-off lump-sum user fee (i.e., a fee which has to be paid only once, regardless of the number of visits). Find the maximum value of the lump-sum user fee that would make the consumer not worse off under this third proposal, compared to the initial situation (i.e., price p^* per visit and no lump-sum payment).

(d) Prove that $C < \Delta S < E$, where ΔS is the solution to part (a), E to part (b) and C to part (c) above.

Question 4

There is a worker who can purchase e units of education, $e \in [0, \infty]$, at cost $\frac{2e^2}{\theta}$. The worker can be of high ability ($\theta = 2$) or low ability ($\theta = 1$), with the worker knowing her own ability (ability is exogenously given). Note that it is less costly for the worker to achieve a particular level of education if she were of high ability than if she were of low ability.

The worker can be hired by a firm paying wage w, and if hired, her marginal product is θ, where θ is her ability. The firm does not know the worker's ability (it knows θ is either 1 or 2), but might be able to infer it after observing her education choice. In particular, if the firm believes the worker is of high ability, it pays wage $w = 2$, while it pays wage $w = 1$ if it believes the worker is of low ability.

The worker chooses e to maximize utility, and if she works for the firm at wage w, then her utility, given e and θ, is $u(w, e \mid \theta) = w - \frac{2e^2}{\theta}$.

(a) Draw the indifference map of the worker in the education-wage space (assume education to be measured along the horizontal axis and wage to be measured along the vertical axis) if she were of high ability. Also draw the indifference map of the worker if she were of low ability. How many times can an indifference curve of a worker who is of low ability intersect an indifference curve of a worker who is of high ability?

(b) Suppose the firm perfectly infers the worker's ability upon observing her education choice. Suppose also the worker chooses $e = 0$ if she is of low ability. Then how much education would she purchase if she were of high ability?

(c) Continue to assume that the firm perfectly infers the worker's ability upon observing her education choice. Suppose, if the worker is of high ability, her education choice matches what you found in part (b). Then show that she would purchase no education if she were of low ability.

Question 5

An individual lives for two periods, 1 and 2, and has lifetime utility function $U(C_1) + \beta U(C_2)$, where C_1 and C_2 are the consumptions of this individual in period 1 and period 2 respectively, and $0 < \beta < 1$ is the subjective discount factor. The following conditions are satisfied: $U'(C) > 0, U''(C) < 0, \lim_{C \to 0} U'(C) = \infty, \lim_{C \to \infty} U'(C) = 0$.

The individual earns an exogenously given income $w > 0$ in period 1 of his life and earns nothing in the second period of his life. But he can lend or borrow freely at an exogenously given rate of interest $r > 0$.

It follows that $C_1 = w - S$ and $C_2 = (1 + r)S$, where S is the savings made by the individual in period 1. Note that C_1 (or S) and C_2 are endogenously determined and the exogenously given parameters of the model are w, r and β.

(a) Suppose $\beta(1 + r) = 1$. Solve for C_1 and C_2 in terms of the exogenously given parameters.

(b) Now remove the restriction that $\beta(1 + r) = 1$. How would savings S be affected if β goes up, ceteris paribus?

(c) Continuing without the restriction that $\beta(1 + r) = 1$, show that savings S goes up as w increases, ceteris paribus. Further show that the increase in S is less than the increase in w.

ISI PEB 2025

Question 1

Consider a monopoly firm facing a market demand function $p = 12 - q$, where p is price and q is quantity. The monopolist has a single plant that can produce an output of q at a cost of $C(q)$, where $C(q) = 2 + \frac{q^2}{2}$, if $q > 0$, and $C(q) = 0$ otherwise.

(a) Find the optimal monopoly output. What is the deadweight loss and social welfare at the optimal monopoly output?

(b) Suppose the monopoly firm can price discriminate perfectly and can also sell in the world market for a constant price of 8. Solve for the optimal monopoly outcome.

(c) Next, suppose the monopoly firm has access to two plants, 1 and 2. Plant i, $i = 1, 2$, can produce an output q_i at a cost of $q_i^2/2$.

i. Solve for the firm's aggregate cost function, i.e., the total cost in case it decides to produce a total output of q using either one or both the plants. Use the aggregate cost function to solve for the optimal monopoly output.

ii. What is the optimal monopoly outcome if the monopoly firm can price discriminate perfectly?

Question 2

An economy consists of two agents 1 and 2 and an initial endowment of Rs 100. A decision has to be made regarding the building of a bridge: $d = 1$ if the bridge is built and $d = 0$ if it is not. The bridge costs Rs 60 to build. If the bridge is built, the remaining Rs 40 is distributed among the two agents; if it is not built, Rs 100 is distributed among the agents. An allocation in the economy consists of a triple (d, x_1, x_2) where $x_1 + x_2 = 40$ if $d = 1$ and $x_1 + x_2 = 100$ if $d = 0$. In each case $x_1, x_2 > 0$. Agent i, $i = 1, 2$, has a valuation $v_i > 0$ for the bridge. Agent i's utility from the allocation (d, x_1, x_2) is $v_i d + x_i$.

(a) Suppose $v_1 = 25$, $v_2 = 45$. Is the allocation $(d = 0, 40, 60)$ Pareto-efficient? Justify your answer.

(b) Suppose $v_1 = 25$, $v_2 = 45$. Is the allocation $(d = 0, 70, 30)$ Pareto-efficient? Justify your answer.

(c) Show that if the allocation $(d = 1, x_1, x_2)$ is Pareto-efficient, then $v_1 + v_2 \geq 60$.

Question 3

(a) An economist collects data from an experiment. Each data point is one of two types: (i) male or (ii) female. The data is collected and sent in three boxes. One box contains data of only male type, another box contains data of only female type, and the third box contains data of both male and female type. When the boxes reach the office of the economist, there are labels on each box.

1. Box 1 - male
2. Box 2 - female
3. Box 3 - male & female

The economist is told that the labels in every box is wrong. The economist asks an MSQE student to sample data from the boxes and figure out the correct labels. What is the minimum number of samples that the student needs to draw to figure out the correct labels of all the boxes? Describe your answer logically by showing which box(es) should be sampled and how many times?

(b) Three students are standing in a straight line. Student 1 is at the front, student 2 is next, and student 3 is at the last position. There are 3 RED hats and 2 BLUE hats. A teacher comes and puts a hat on each student. Suppose each student only sees the colour of the hats of students in front of her, but not her own hat or the hats of students behind her. So, student 3 sees the colour of the hats of student 1 and student 2; student 2 sees the colour of the hat of student 1; and student 1 does not see the colour of anyone's hat. Starting with student 3, followed by student 2, and finally student 1, each student is asked if she knows the color of her own hat. The students can either answer yes or no. Assume students answer truthfully, and answer of each student is revealed to all students.

   i. What is the probability that student 3 says "no"?
   ii. Suppose student 3 says "no." What is the probability that student 2 says "no"? Note that student 2 knows the answer of student 3.
   iii. Suppose student 3 and student 2 both say "no." What is the probability that student 1 says "no"? Note that student 1 knows the answers of student 3 and student 2.

Question 4

(a) How many real solutions does the following equation have?

$$(x^2 - 5x + 6)^{(x^2-7x+12)} = 1$$

(b) A function $f : \mathbb{R} \to \mathbb{R}$ is called quasi-convex if for every $x, y \in \mathbb{R}$ and every $\lambda \in (0, 1]$, we have

$$f(\lambda x + (1 - \lambda)y) \leq \max\{f(x), f(y)\}.$$

   i. Show that a convex function is quasi-convex.
   ii. Show that a non-decreasing function is quasi-convex.
   Note that f is non-decreasing if $x > y$ implies $f(x) \geq f(y)$.

Question 5

(a) Suppose that utility function is given by $u = \ln c + b\frac{(1-l)^{1-\gamma}}{1-\gamma}$ where c and l represent consumption and labour supply, and parameters b and γ are positive. Further, the wage rate per unit of labour supply is given by w. Agents consume out of their labour income which forms their budget constraint. If agents maximise utility subject to their budget constraint, how does labor supply depend on the real wage rate w. Clearly show all the derivations and explain the result.

(b) Now consider that agents live for two periods and they have the following utility function: $u = \ln c_1 + 2b\sqrt{(1 - l_1)} + \beta\left[\ln c_2 + 2b\sqrt{(1 - l_2)}\right]$ where c_i, l_i represent consumption and labour supply in period $i = 1, 2$ respectively, the parameter β is positive while the rest of the notation is the same as in part (a) above. Further, any consumption made or income earned in the second period is discounted at the rate, $1 + r$. Agents maximise their utility subject to the life-time budget constraint. Further, leisure in period $i = 1, 2$ is denoted by $q_i = 1 - l_i$.

i. Find out the expression for the ratio of the optimal $\frac{q_1}{q_2}$ in terms of the wage ratio, $\frac{w_2}{w_1}$, and other parameters of the model. How does l_1 vary with $\frac{w_2}{w_1}$? Explain the result.

ii. Let q^* be the ratio of optimal q_1 to q_2, that is, $\frac{1-l_1}{1-l_2} = q^*$ and $\frac{w_2}{w_1} = w^*$. Calculate the elasticity of q^* with respect to w^*. Interpret your result in a couple of sentences.

ISI PEB 2026

Question 1

For any natural number m, let $m! = m \times (m-1) \times \cdots \times 2 \times 1$. Let $(a_k)_{k=1}^{\infty}$ be a sequence of positive real numbers. Suppose that

$$\sum_{k=1}^{\infty} \frac{a_{k+1}}{a_k} \leq 1.$$

Show that for all natural numbers $n \geq 2$,

$$a_n \leq \frac{1}{(n-1)!}.$$

Question 2

Consider for each $t > 0$, the system of two linear equations in two variables

$$(1+t^2)x + ty = 0,$$
$$t^2 x + e^{-t} y = 0.$$

Show that there exists a number $t_0 > 0$ such that at $t = t_0$, the set of solutions (x, y) of the above set of equations will form a one-dimensional subspace of $\mathbb{R}^2$.

Question 3

Let $T > 0$. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function with period T, that is,

$$f(x+T) = f(x)$$

for all $x \in \mathbb{R}$. Show that f is differentiable.

Question 4

An experiment has N outcomes, say $\omega_1, \omega_2, \ldots, \omega_N$. Suppose ω_{j+1} is twice as likely as ω_j where $j \in \{1, 2, \ldots, N-1\}$. Let $A_k = \{\omega_1, \omega_2, \ldots, \omega_k\}$. Find $P(A_k)$.

Question 5

There are M students of which K are female and $M-K$ are male, $K \in \{1, \ldots, M-1\}$. A debate committee comprising of $n \in \{1, \ldots, M-1\}$ students has to be formed. An urn with the unique roll numbers of all students is created to ensure that the selection process is randomized and unbiased. Roll numbers are drawn one by one from the urn following the rule that once a roll number is selected, it is withdrawn permanently from the urn.

(a) Find the probability that the j^{th} number drawn from the urn is that of a female student given that k female students have already been selected into the committee.

(b) Find the probability that the j^{th} number drawn from the urn is that of a female student given that there are a total of k female students in the committee.

Question 6

A profit maximizing firm possesses a linear technology which can convert 1 unit of labour input into 1 unit of output. Labour is available at wage $w > 0$ per unit. The firm, a monopoly, faces an inverse demand function $p(q)$, with $p(0) > w$, $\lim_{q\to\infty} p(q) = 0$, $\lim_{q\to\infty} qp(q) = 0$, $p'(q) < 0$ and $p''(q) \leq 0$. The firm first hires labour, then executes production, following which it sells output to consumers and collects revenue.

(a) Suppose labour can be paid wages after the collection of revenue. What is the firm's optimal output?

(b) Suppose now labour must be paid wages prior to the collection of revenue.

i. Suppose further that the firm has no cash in hand to pay wages upfront, but can borrow an amount B prior to production to pay wages upfront, with repayment due after the collection of revenue. The net interest rate on any such loan is $r > 0$. What is the firm's optimal output?

ii. Suppose instead further that the firm cannot borrow, but it has cash $C \geq 0$ available with it which can be used to pay wages upfront. What is the firm's optimal output?

Question 7

A household lives for two periods, $t = 1, 2$. In each period it chooses consumption C_t and labor supply N_t. It also chooses savings S in period 1, which pays gross return $(1 + r)$ in period 2. The household earns wage income $w_t N_t$ and receives government transfers T_t in period t, where w is wage rate.

Preferences are

$$U = \ln C_1 + \ln(1 - N_1) + \beta[\ln C_2 + \ln(1 - N_2)], \quad \beta \in (0, 1).$$

The household faces budget constraints

$$C_1 + S = w_1 N_1 + T_1$$
$$C_2 = (1 + r)S + w_2 N_2 + T_2.$$

Assume the government satisfies the present-value transfer constraint

$$\overline{T} \equiv T_1 + \frac{T_2}{1 + r}$$

where $\overline{T}$ is fixed.

(a) Solve for the household's optimal consumption allocations (C_1, C_2).

(b) Does optimal consumption depend on the timing of transfers (T_1, T_2) or only on their present value $\overline{T}$?

(c) Now suppose the household evaluates period-2 utility with an additional present-bias factor $\delta \in (0, 1)$:

$$U = \ln C_1 + \ln(1 - N_1) + \delta\beta[\ln C_2 + \ln(1 - N_2)].$$

The household cannot commit: in period 1 it chooses (C_1, N_1, S), and in period 2 it chooses (C_2, N_2) taking savings S as given. Solve for the optimal consumption allocations (C_1, C_2).